Optimum Dimensions of Hydraulic Structures and Foundation Using Genetic Algorithm coupled with Artificial Neural Network

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ABSTRACT

A model using the artificial neural networks and genetic algorithm technique is developed for obtaining optimum dimensions of the foundation length and protections of small hydraulic structures. The procedure involves optimizing an objective function comprising a weighted summation of the state variables. The decision variables considered in the optimization are the upstream and downstream cutoffs lengths and their angles of inclination, the foundation length, and the length of the downstream soil protection. These were obtained for a given maximum difference in head, depth of impervious layer and degree of anisotropy. The optimization carried out is subjected to constraints that ensure a safe structure against the uplift pressure force and sufficient protection length at the downstream side of the structure to overcome an excessive exit gradient. The Geo-studio software was used to analyze 1200 different cases. For each case the length of protection (L) and volume of structure (V) required to satisfy the safety factors mentioned previously were estimated for the input values, namely, the upstream cutoff depth (S_1), the downstream cutoff depth (S_2), the foundation width (B), the angle of inclination of the upstream cutoff (Θ_1) and the angle of inclination of the downstream cutoff (Θ_2), H (difference head), k_r (degree of anisotropy) and D (depth of impervious layer). An ANN model was developed and verified using these cases input-output sets as its data base. A MatLAB code was written to perform a genetic algorithm optimization modeling coupled with this ANN model using a formulated optimization model. A sensitivity analysis was done for selecting the cross-over probability, the mutation probability and level, the number of population, the position of the crossover and the weights distribution for all the terms of the objective function. Results indicate that the most factors that affects the optimum solution is the number of population required. The minimum value that gives stable global optimum solution of this parameter is (30000) while other variables have little effect on the optimum solution.

Key words: inclined cutoff, optimization, genetic algorithm, artificial neural networks, uplift pressure, exit gradient, factor of safety.

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الخلاصة

تناول هذا البحث بناء نموذج الامثلية بأستخدام برنامج الماثلاب الذي استخدم لتطبيق تقنية الجينات الوراثية وتقنية الشبكات العصبية الصناعية للوصول الى التصميم الامثل لأساس منشأ هيدروليكي صغير. هذا النموذج تم بناءه لايجاد الأبعاد المثلى للقواطع الاساس في كل من المقدم و المؤخر وكذلك ايجاد ميلان تلك الركائز وطول الارضية الاساس و طول الحماية المطلوبة في المؤخر في المنشات الهيدروليكية. تم ايجاد هذه الابعاد لقيم معطات لكل من اعلى فرق للشحنة بين مقدم و مؤخر المنشأ، و لعمق الطبقة الصماء و درجة التباين في قيم خواص التربة مع الاتجاه. دالة الهدف التي تم ايجاد القيم الصغرى لها هي دالة الكلفة بمعاملات كلفة نسبية. اما المحددات المستخدمة في النموذج فهي معاملات الامان ضد ضغط الرفع و غليان التربة.
تم نمذجة عدة حالات تحديدا 1200حالة باستخدام برنامج Geo-studio. في هذه النمذجة تم اعتبار التربة متجانسة و ذات تباين مع الاتجاه. لكل حالة تم حساب طول الحماية L و حجم المنشاء V المطلوبة لتحقيق معاملات الامان المشار اليها أعلاه.
تم استخدام البيانات الخاصة بالحالات اعلاه لبناء نموذج شبكات العصبية لحساب L و V لقيم معطات من عمق القاطع في المقدم (S_1)، عمق القاطع في المؤخر(S_2) ، ميل القاطع في المقدم (Θ_1)، عمق القاطع في المؤخر (Θ_2)، فرق الشحنة بين المقدم و المؤخر(H)، طول الارضية(B) ، عمق طبقة الصماء (D) و درجة التباين(k_x/k_y) مع الاتجاه في خواص التربة.
تم كتابة برنامج Matlab لنموذج الجينات الوراثية يستخدم نموذج شبكات العصبية المشار اليه اعلاه. باستخدام هذا النموذج تم ايجاد الحل الامثل لبعض الحالات المختارة و تم مقارنتها بالنتائج المناضرة التي تم الحصول عليها باستخدام برنامج Geo-studio كانت نتائج النموذجين متقاربة.
تم اختبار حساسية التحليل لمعدل الانتقال (pc) ، معامل الطفرة (pm) ، مستوى الطفرة (ML)، حجم السكان المولد (np)، موقع الـ(cross overing) ومعاملات الكلفة النسبية لدالة الكلفة. بينت النتائج بأن حجم السكان المولد (np) من اهم معاملات نماذج الموروثة الجينية والتي تؤثر على القيمة المثلى للمتغيرات المذكورة سابقا. وقد وجد بأن حجم السكان المولد (np) الذي يعطي الحل المستقر ليس اقل من (30000).
الكلمات الرئيسية: جدار قاطع مائل، الحل الامثل، الموروثة الجينية، شبكة عصبية صناعية، ضغط الرفع، انحدار المخرج، معامل الامان.

1. INTRODUCTION

Hydraulic structures foundation proper design has a vital role on the safety of such structures. The most common failures of these structures are either due to uplift pressure forces, and/or due to the failure of the soil at the downstream side due to piping effect, which consequently results into a tilting failure of the whole structure. Provision usually provided to avoid such failures are the use of upstream and downstream cutoffs, a protection of suitable length at the downstream side, and adequate volume of the super structure in order to achieve the required factors of safeties against these failures. The required factors of safeties against uplift pressure and piping failures are usually assigned according to the recommendations of authorized codes and pioneers experts and scientist of the design of these structures such as **Koshla**, **1954**. Different attempts were found on the literature that focuses on the role and effectiveness of using cutoffs and protections that assure the safety against such failures. Recently many optimization models were developed to decide the suitable dimensions of these cutoffs and protections.

Many of researches were conducted assuming the soil beneath the structure as homogeneous and isotropic media. The real case is different, since the soil is heterogeneous and anisotropic. However some researches were conducted assuming the soil is non-homogeneous but isotropic. The present study represents an attempt to obtain an optimization model to find the optimum dimension of the foundation length, cutoffs, and the downstream protection, by considering the soil as anisotropic and using inclined cutoffs. The use of inclined cutoffs with an isotropy is expected to be more reasonable since for anisotropic media the flow lines are affected by the degree of anisotropy, and a certain inclination may be the optimum solution that minimizes the effect of this anisotropy. However the inclination angles of the cutoffs are included in addition to their lengths, foundation width, and length of protection and volume of the super structure. This model is a genetic algorithm coupled with artificial neural networks.

Historically batter piles (inclined piles) were used to resist lateral forces and inclined forces especially in water front structures. The forces on these structures are axial loads due to self-weight of the superstructure and lateral loads due to water and seepage water. However, due to poor performance in recent time, plumb piles (vertical piles) are now the system of choice. Nevertheless, there are situations where batter piles are desirable, for example, where the new structure has to be compatible with an existing batter pile structure or has high service-level lateral loading conditions such as ship mooring (**Harn**, **2004**). In addition, certain difficulties might be experienced in driving the sheet piles vertically downwards (**Ram** and **Vaidhianathan**, **1940**).

Most of the earlier studies were concerned only with one embedded inclined sheet pile. However, limited literature is available concerning the use of two inclined sheet piles. The calculated exit gradient values, flow rates, and uplift pressure were proved to be affected by changing the slope of the angle of the sheet pile and varying the soil properties. Limited literature is available for seepage through pervious medium beneath hydraulic structures with inclined cut-offs as a control device.

Ram and **Vaidhianathan**, **1940** determined the distribution of uplift pressure under weirs with a single sheet pile inclined to the floor. **Siva** and **Basu**, **1976** developed an analytical solution, making use of the Schwartz-Christoffel transformation for determining the seepage characteristics for the problem of flow under a weir having two unequal sheet piles at the ends and embedded in an anisotropic porous medium of finite thickness. **Al-Suhaili et al.**, **1988** investigated a direct mathematical approach to obtain the exit gradient variation downstream of all types of structures for both infinite and finite porous media for design purpose. **Ilyinsky** and **Kacimov**, **1992** investigated an analytical estimation of groundwater flow around cutoff walls and into interceptor trenches. **Griffiths** and **Fenton**, **1993** studied the effect of stochastic soil permeability on confined seepage occurring beneath water retaining structures. Random field concepts were used to generate permeability fields having predefined mean, standard deviation and correlation structure. **Prabhata et al.**, **1997** studied the effectiveness of multiple sheet piles in weir design. Based on cost optimization (expressed by **Swamee et al.**, **1996**), the researchers present a theoretical justification of the viewpoint given by **Sowers** and **Sally**, **1960**. **Rajashree** and **Sitharam**, **2001** studied the static and cyclic lateral responses of vertical and batter piles based on a newly developed nonlinear finite element code using hyperbolic and modified hyperbolic relations to represent the nonlinear behavior of soil. **Hassan**, **2002** investigated the optimum design of the control devices for safe seepage under hydraulic structures. **Al-Joubori**, **2002** established a model of seepage below hydraulic structure with two vertical cutoffs by using the Finite Element technique combined with random field concepts for the generation of soil permeability properties with specified mean, variance and spatial correlation length. **Tayfur et al.**, **2005** investigated a Finite Element Method and Artificial Neural Network Models for Flow through Jeziorsko earthfill dam in Poland. **Ersayin, 2006** used the artificial neural networks to study seepage through the body of an earth fill dam using MATLAB 6.0 Neural Network Toolbox. **Alsenousi** and **Mohamed, 2008** developed a two dimensional finite element model to analyze seepage flow beneath a dam with an inclined sheet pile. **Chen Y et al.**, **2008** performed a numerical solution for seepage problems with complex drainage systems. A numerical solution based on the Finite Element Method combining the substructure technique with a variation inequality formulation of Signorini's type was proposed to solve these problems. **Karim**, **2011** developed a Genetic Algorithm model coupled with Artificial Neural Network model to find the optimal values of upstream, downstream cutoff lengths, length of foundation and length of downstream protection required for a hydraulic structure. **Al-Suhaili**, **2009** obtained the exit gradient variation along the downstream side for an inclined sheet pile using analytical solution. **Miao**, **et al.**, **2011** Predicted seepage of earth dams using neural network and genetic algorithm for levenberg-marquardt (GA-LM). **Singh**, **2011** investigated optimal hydraulic structures profiles under uncertain seepage head. He had formulated an optimization problem using Genetic Algorithm model to obtain the optimum structural dimensions that minimize the cost as well as satisfy the exit gradient criteria. **Arun** and **Lakshmi**, **2011** obtained Closed-form theoretical solutions for steady seepage below a horizontal impervious apron with equal end cutoffs using Schwarz-Christoffel transformation.

Arslan and **Mohammad**, **2011** conducted an experimental and theoretical study for piezometric head distribution under hydraulic structures to test the effect of upstream, intermediate and downstream sheet piles inclination, and then the optimum case of the uplift pressure reduction was found. The solution was developed using the Schwarz-Christoffel transformation. **Ijam**, **2011** used an analytical solution to obtained seepage flow below a dam structure with inclined cutoff located anywhere along the base of the dam. **Al-Saadi**, **et al.**, **2011** investigated the effect of cut-off inclination angle on exit gradient and uplift pressure head under hydraulic structure. The optimum location and angle of inclination of cut-off have been also determined. This problem is solved using the finite element method by using (ANSYS 11.0). **Arun** and **Lakshmi**, **2012** obtained the closed-form solution to the problem of finite depth seepage under an impervious flat apron with equal end cutoffs, with a downstream step, using the conformal transformations.

From the above studies, it appears that no solution was available in the literature to develop an optimization model which will eliminate the difficulty faced by the designers of small structures, in deciding the proper dimension length and angle of inclination of the cutoffs. An investigation of the proper design of the structure with floor having two inclined cutoffs was, therefore, made.

In this research, a model was developed to optimize the dimensions of the structure foundation having two inclined cutoffs. It is evident from the present study that these optimum dimensions can minimize the formulated relative cost objective function. The GEO-SLOP, SEEP/W 2007 (version 7.10 build 4143) model was used to establish a data base which is used later to develop an artificial neural network (ANN) model that relates the relevant input output variables of the problem. Finally this ANN model was coupled with a genetic algorithm model, to optimize the dimensions mentioned above. The ANN model provides the direct estimation of the required outputs which are required for the genetic algorithm model.

2. FORMULATION OF THE OPTIMIZATION MODEL

The most critical design of a hydraulic structure is the foundation design. The required dimensions for the design of the foundation are the length of floor (B), depth of upstream cutoff (S_1), depth of downstream cutoff (S_2), angle of inclination of these cutoffs (Θ_1, Θ_2), length of protection at the downstream side against exit gradient (L), and the volume of superstructure (V) for a given head difference (H), depth of impervious layer (D), and given soil properties underneath the structure, horizontal permeability k_x, and vertical permeability k_y. **Fig. 1** shows these dimensions for a typical hydraulic structure.

The values of (S_1, S_2, L, and V) are affected by the maximum expected difference in head between the upstream and downstream sides of the hydraulic structure (H) and the soil strata properties (k_x and k_y). The most critical failures that may occur for such structures are either due to the uplift pressure or due to erosion of the downstream side, when the hydraulic gradient exceeds the critical exit gradient. The designer can control these failures by providing the recommended factors of safety against both uplift pressure and exit gradient failures. The controlling process was done by selecting the dimensions of S_1, S_2, B,Θ_1, Θ_2 and L for a given (H), (D) and (k_x/k_y). It is better to select optimum dimensions; the following objective function of such a problem could be introduced.

The cost objective function involves the cost of both floor and any control device and can be used to achieve the optimum dimensions of the hydraulic structure. Such a function is formulated as follows:

$$F(B, S_1, S_2, L, \Theta_1, \Theta_2) = C_B*B + C_{S1}*S_1 + C_{S2}*S_2 + C_L*L + C_V*V \quad (1)$$

F: cost function that should be minimized.
C_{S1}, C_{S1}, C_B, C_L, C_V: relative weight (cost) of each dimension; the weight should satisfy the following requirement: $0 \leq C \leq 1$ and $\sum C = 1$
B, S_1, S_2, Θ_1, Θ_2: dimensions and inclination angles of the hypothetical case study (defined in fig.1).
L and V: length of protection downstream side of the structure and volume of super structure respectively.
This function is subjected to the following Constraints:

$$F.O.S_{uplift} = \frac{\gamma_c V}{uplift\ force} \geq 2 \quad (2)$$

$$\frac{icr}{i} \geq 3 \quad (3)$$

Where:
F.O.S uplift: factor of safety against uplift pressure
γ_c: Concrete weight density (24.5 KN/m^3)
V: Volume of concrete of the super structure
i_{cr}: Critical exit gradient (equal to one)
i: The computed exit gradient at the downstream side of the structure.
Additional constraints were also adopted to allow for much control of the decision variables as follows:

$$\begin{pmatrix} S_{1min} \leq S_1 \leq S_{1m} \\ S_{2min} \leq S_2 \leq S_{2max} \\ B_{min} \leq B \leq B_{max} \\ \Theta_{1min} \leq \Theta_1 \leq \Theta_{1max} \\ \Theta_{2min} \leq \Theta_2 \leq \Theta_{2max} \end{pmatrix} \quad (4)$$

The Genetic Algorithm was used to solve the optimization problem mentioned above. The optimization involves computation of state variables such as B, S_1, S_2, Θ_1, Θ_2, L and V. The computation of L and V was done by using the developed ANN model that will be shown thereafter.

3. THE GEO-STUDIO MODEL

Any optimization method needs an explicit relationship between the input-output variables and its variables partial derivatives. This relationship is either representing the objective function as the output or representing output variables that used to estimate the objective function. Part of these input and output variables may be a decision variables, and the other are of non decision variables (given). The genetic algorithm method requires only the estimation of the objective function and do not requires the derivatives. For the problem under consideration there exists no direct relationship between the input variables (H,D, K_x and k_y,S_1,S_2, Θ_1, Θ_2, and B) and their respective output variables (L and V),that could be used to estimate the objective function, given by Eq.(1). Hence, as the genetic algorithm model start the solution by generating randomly a large number of feasible solution set for S_1,S_2, Θ_1, Θ_2, and B), for a given H,D, K_x, and K_y. For each generated set of these variables, the values of L and V should be estimated that satisfy Eqs. (2) and (3). These L and V requires the solution of the seepage flow equation using finite differences or finite elements method to obtain the head distribution in the flow field beneath the structure, hence obtaining the uplift force and the exit gradient, which allows the estimation of L and V. The methodology adopted here is to develop a direct relationship between the input and an output variable of the problem under consideration, and also to develop a representative data base of these variables using the Geo-studio software, which adopt the finite element solution of the seepage problem. This software provides high level of accuracy and extensive graphical representations of the results. Then this data base was used to obtain a simple direct relationship between the input and output variables, that can be used easily in the genetic algorithm optimization model.

The data base mentioned above was performed by modeling 1200 different cases using the Geo-studio software. Each case has different selected values of S1, S_2, H, B, D, k_r, Θ_1 and Θ_2. The values of exit gradient, seepage flow beneath the structure, and the uplift pressure under the structure were calculated. These results can be used to estimate the volume of concrete (V) and the length of the downstream protection (L) such that the constraints of Eqs. (2) and (3) were achieved respectively.

The selected ranges of each variable S1, S_2, H, B, D, K_r, Θ_1 and Θ_2 are as follows:

S_1,S_2 (0.5-4)m steps 0.5, H (6-16)m steps 2, B (6-16)m steps 2, D (10-12)m steps 2, K_r(1-8) and Θ_1, Θ_2 (50^o-130^o)

Fig. 2 shows the structure for one of the cases with the discretization process. This figure shows also the uplift pressure distribution beneath the structure, flow lines and equipotential lines. **Fig. 3** shows the distribution of the exit gradient along the downstream side of the structure. The required length of protection can be estimated using this curve and Eq.(3).**Table 1** shows the results of some cases analyzed using the Geo-studio models.

4. DEVELOPMENT OF THE ARTIFICIAL NEURAL NETWORK (ANN) MODEL

The results of L and V for the (1200) cases were used for building an ANN model capable of estimating L and V as output variables using S_1, S_2, H, B, D, k_r, Θ_1 and Θ_2 as input variables.

In order to obtain this model, the SPSS software (Statistical Procedure for Social Science, version 20.0) was used. The ANN model comprised eight neurons in the input layer where these neurons represent the input variables namely S_1, S_2, H, B, D, k_r, Θ_1 and Θ_2. Two neurons were selected for the output layer which represents the output variables (L and V). The network was built with one hidden layer having nine neurons. The initial learning rate and the initial momentum term were chosen equal to (0.4) and (0.9) respectively. The selected activation functions are the hyperbolic tangent for the hidden layer and the identity for the output layer.

To build the ANN model, many run trials were performed, in each one the software parameters were changed as follows:

- Selection of the percentages of division of the data into training, testing, and validation subsets.
- The selection of the division method either blocked, stripped, or random.
- Testing the proper number of nodes in the hidden layer.
- Changing the learning rate and momentum factor.
- The selection of the best ANN model was achieved according to the smallest error and the highest correlation coefficient of the predicted and observed outputs.

Table 2 represents the best data division and **Fig. 4** shows the architecture of the ANN network. **Table 3** shows the bias and weight matrices for the input and hidden layers. **Fig. 5** and **Fig. 6** show the comparison between the predicted and observed values of L and V, respectively. The results of the ANN model indicated high correlation coefficients between the observed and predicted values of L and V as r_L= 97.5% and r_V= 99.4% respectively. Even though the ANN modeling procedure involves the subdivision of the data into subsets as mentioned above, and uses the first two subsets for model parameter estimation, and the third set for validation, further verification was performed herein.

A MatLAB (R2008a) code was written to perform the Algorithm shown in the steps (mentioned in appendix A), used to estimate (L and V) for different values of (S_1, S_2, H, B, D, K_r, Θ_1 and Θ_2).

Table 4 shows the comparison of the values of L and V estimated using both Geo-studio and ANN models, for 12 cases that are not involved in the data base of 1200 case mentioned above. These results indicate the capability of the ANN model to produce acceptable results.

5. OPTIMIZATION USING GENETIC ALGORITHM (GA) MODEL

A MatLAB code was written for the solution of the optimization model formulated above, using genetic Algorithm method as the steps shown in the appendix B. The following values were adopted for the constraints shown in eq.(4), S_{1min}=0.5m, S_{1max} = 4m, S_{2min} = 0.5m, S_{2max} = 4m, B_{min} = H, B_{max} = 3H. In any application of the genetic algorithm there exists different parameter that affects the optimum solution. The proper value of each of these parameters should be obtained for the specific application. For the application under study the following analysis was performed to obtain the suitable value of each of these parameters.

The first parameter that should be obtained is the initial number of solutions that are generated randomly, usually called initial population np. Generating low number of random solution may result in an unstable solution, i.e., each run gives different optimum solution. In order to arrive to a stable optimum solution there always exist a minimum np values that produces almost the same optimum solution. **Fig. 7** shows the obtained optimum objective function of three runs for different values of np, and given crossover position k=4, and 100% crossover Pc=1. Results indicate that the required np value for stable solution is 30000.

The second parameter is the number of iterations required to obtain stable solution. That number of iteration involves the number of the crossover operation performed. It was found the required number of iterations which gives stable solution is 3. The application for cases of different crossover position indicates the same values of np and number of iteration required for stable solution; hence these values are fixed for the further analysis.

The third parameter that may have effect on the obtained optimum solution is the cross over position k. A sensitivity analysis was done for selecting this position which can be defined as corresponding points at which the two mating pairs are each cut once, and an arbitrary substrings exchange with probability pc. Length and position of these substrings are chosen at random, but are identical for both pairs. The crossover may involve more than one cut point. These tests were done by taking different positions as k=1, k=2, k=4 and k=7. **Fig. 7** presents the result of optimum solution with the crossover position value equal to (k=4). Others crossing site are presented in **Fig. 8**, **Fig.9** and **Fig.10**. On analyzing the results shown in the above figures for different values of crossover position, it is noticed that there is no high difference in the obtained the optimum solution. However for 2-point crossover (**Fig.8**) the least value of f(x) was obtained.

Sensitivity analysis was also done for the parameter pc (the probability of crossover) in order to find the effect of this parameter on the results obtained by the model. For this test, the number of population size was fixed at 30000. **Table 5** illustrates that (pc) had little effect on the solution.

The above four parameters are concerned with the genetic algorithm method. The following analysis includes the effect of the different weight values assigned to the objective function variables on the optimum solution of the problem under study. The relative weights for the above analysis were chosen as C_{S1}=0.2, C_{S2}=0.2, C_B=0.1, C_L=0.1, C_V=0.4. In order to find the effect of the relative weight on the results obtained by GA model, the sensitivity analysis on this parameter was also done. Three different weights distribution as (C_{S1}=0.2, C_{S2}=0.2, C_B=0.2, C_L=0.2, C_V=0.2), (C_{S1}=0.1, C_{S2}=0.1, C_B=0.3, C_L=0.1, C_V=0.4) and (C_{S1}=0.1, C_{S2}=0.1, C_B=0.1, C_L=0.1, C_V=0.6) were examined. The results of this analysis are presented in **Fig.11**, **Fig.12** and **Fig.13** respectively. The results of using an equal weight distribution for all the dimensions of the objective function (S_1, S_2, B, L and V) show that the value of f(x) decreases to almost half of its initial value as indicated in **Fig.11**. While for the second weight distribution the value of f(x) remained unchanged. This result may be expected since the weight C_V reduces to a half of its initial value for the case of equal weights. The volume of the structure (V) is the most significant variable that affects the objective function. Therefore reducing the weight of (C_V) to half will affects the value of the objective function to reach half of its initial value.

The most important improvement that can be made on the obtained optimum solution is mutation which involves the modification of the value of each gene of a solution with some probability pm. Therefore some optimal solutions were chosen to apply this improvement. During the runs, the probability of mutation (pm) of 0.1 was used (10%). For this test, the first best three runs were chosen from **Table 7**. The MatLAB code was written for doing the mutation process. From the results summarized in table 8, it can be seen that no big changes in the values of the objective function were observed for all of the three runs. This shows that the mutation has a little effect on the optimum result.

6. CONCLUSIONS

From the present work, the following conclusions could be obtained:

1) The artificial neural network model, found to be efficient in obtaining the values of the length of protection in the downstream side (L) and the volume required for superstructure (V) with a correlation coefficients 97.5% and 99.4% respectively. The required number of hidden nodes was 9 with one hidden layer. Among different types of the activation functions of the hidden and output layer tried, the hyperbolic tangent and the identity functions were found to be the most suitable for the hidden and output layer respectively. The best learning rate and momentum term found for the network are 0.4 and 0.9 respectively.
2) In the genetic algorithm model application for the problem understudy, indicated that as the size of population of the solutions initially generated randomly increased, the differences in the obtained optimal solution for different runs of the same input values are decreased. These differences are insignificant when the size of population 30000, i.e. the same optimum objective function and decision variables for all the runs, which is the required size of population for stable solution.
3) The stable solution obtained for the size of population of 30000, requires 3 iterations of the crossing over processes. Further iteration does not improve the optimum solution, i.e. the global optimum was reduced using this number of iteration.
4) The genetic algorithm model indicates that the values of probability of crossing-over, probability of mutation and mutation level have little effect on the obtained optimal solutions for the problem studied.
5) Selecting different positions of the crossover using an integer position (k=1, 2, 4 and 7) reveals that there are no large differences in the optimum solution. However, for k=2, the least value of {f(x)} was obtained.
6) The obtained optimum solution using the genetic algorithm model is robust, i.e. each run give different solution, and however, a slight difference was obtained for the decision variables for most of the solutions. Hence, the designer should select the solution that gives the minimum objective function {f(x)}.
7) The relative weight distribution of the objective function variables was found to have high affection the optimum solution. Using an equal weight distribution for all dimensions of the objective function (S_1, S_2, B, L and V), the value of {f(x)} decreases to almost half of its initial value. Hence, the designer should carefully choose a proper weight for each dimension mentioned above.

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Appendix A: The Steps of MatLAB Code for the ANN Model

1. Structure data input:

- Enter the maximum expected difference in head between upstream and downstream sides (H in meters),
- Enter the value of impervious layer depth (D in meter)
- Enter kr = k_x/k_y ratio of horizontal to vertical permeability.
- Enter the floor length (B maximum in meters),
- Enter the length of upstream cutoff (S_1 in meter) < depth of impervious layer (D),
- Enter the length of upstream cutoff length (S_2in meter),
- Enter the values of Θ_1 and Θ_2.

2. Enter the model parameters matrices from Ann model (table 3):

-0.084	0.054	0.632	0.58	0.555	0.037	-0.045	0.062
0.041	0.218	0.1	0.124	-0.023	0.064	-0.155	-0.22
0.059	0.349	-0.572	-0.112	0.427	-0.128	0.011	-0.193
0.022	-0.102	0.731	-0.137	-0.212	-0.046	-0.022	0.124
-0.052	-0.091	0.238	1.022	0.183	0.062	0.007	0.246
-0.086	0.032	0.569	-0.165	0.655	-1.467	-0.037	-0.128
0.056	-0.261	-0.009	-0.187	0.275	1.187	0.252	0.186
-0.032	0.195	-0.442	-0.106	-0.434	0.366	-0.095	0.26
-0.345	0.325	-0.037	0.155	0.421	0.382	0.208	-0.104

$V_{o\ bias\ (9\ x\ 1)}= \begin{bmatrix} -1.506 \\ -0.552 \\ -1.345 \\ 0.596 \\ 0.17 \\ -1.826 \\ -0.233 \\ 0.358 \\ 0.277 \end{bmatrix}$ $V_{(8\ x\ 9)}=$

$W_{o\ bias\ (2\ x\ 1)}= \begin{bmatrix} 0.39 \\ 1.096 \end{bmatrix}$

$W_{(9\ x\ 2)}= \begin{bmatrix} 0.212 & 1.354 \\ 0.008 & 0.747 \\ -1.801 & 0.055 \\ 0.492 & 0.795 \\ -0.478 & 0.373 \\ 2.208 & 0.047 \\ -0.432 & -0.142 \\ -0.269 & -0.141 \\ 0.197 & 0.315 \end{bmatrix}$

3. Rescaling the eight input variables by using the standardization method:

$$Xs=(X-mean_x)/sd_x \quad (5)$$

Where:

$X_{(8\ x\ 1)}= \begin{bmatrix} S1 \\ S2 \\ H \\ B \\ D \\ Kr \\ \theta 1 \\ \theta 2 \end{bmatrix}$

4. Find the matrix Zin $_{(9x1)}$

$$Z_{in\ (9x1)}=V_{o\ bias(9x1)}+V^T_{(8x9)} * Xs_{(8x1)} \quad (6)$$

5. Find $Zin_{\ (9x1)}=tansh\ (Z_{in(9x1)})$ (7)

6. Find $y_{(2x1)}= W_{o\ bias(2x1)}+W^T_{(9x2)} * Z_{(9x1)}$ (8)

7. Find $y_{(2x1)}=yin_{(2x1)}$ (9)

8. Find $L=y_1*sd_L+Mean_L$ (10)

$$V= y_2*sd_V+Mean_V \quad (11)$$

Appendix B: Genetic algorithm Solution steps of the Optimization model

A MatLAB (R2008a) code was written to perform the Genetic Algorithm model by using the following steps:

1. Structure data input:
   - Enter the maximum expected difference in head between upstream and downstream sides (H in meters),
   - Enter value of impervious layer depth (D in meter)
   - Enter $k_r = k_x / k_y$ ratio of horizontal to vertical permeability.
   - Enter the relative weights of the objective function
     - Cs_1 : percent cost of upstream cutoff (S_1)
     - Cs_2 : percent cost of downstream cutoff (S_2)
     - C_B : percent cost of the foundation (B)
     - C_L : percent cost of the length of protection (L)
     - C_V : percent cost of the volume of the structure (V)
   - Enter the number of decision variables (nd)
   - Enter the model parameters from ANN
     - n : number of input variables.
     - P : number of hidden nodes.
     - m : number of output variables.
     - Enter the weight matrices V_{obias}, V, W_{obias} and W
   - Enter the number of iterations (nit)
   - Enter the number of population (np)
   - Enter the cross-over probability (pc)
2. Generate random solution of np for each of the decision variable (S_1, S_2, B, Θ_1 and Θ_2).
3. Change the generated solutions to the ranges of each variable (eq.4)
4. Transform of the input variables to the standardized form
5. Calculate (L) and (V) by using the ANN model.
6. Calculate the value of the objective function using eq. (1):
7. Sort values of F(x) in ascending order
8. Select an individual strings according to their objective function values (fitness function) and copied them into the mating pool. The number of pairs to be cross-over (NOCC) will be:

$$NOCC=\left|\frac{np*pc}{2}\right| \quad (12)$$

9. Make cross-over where each pair of strings undergoes crossing over by selecting the position of the cross-over along the string uniformly. The resulting cross-over yields two new strings (offspring) as a result, the new population (npao)will be :

$$npao = np+8*NOCC \quad (13)$$

10. Find the value of F(X) for the new population (npoa) after finding L and V for them, sort in ascending order, and eliminate the last cases of new population (npao/2). The new population is then used in the next iteration of the algorithm.
11. Go to step (8) to make another iteration.

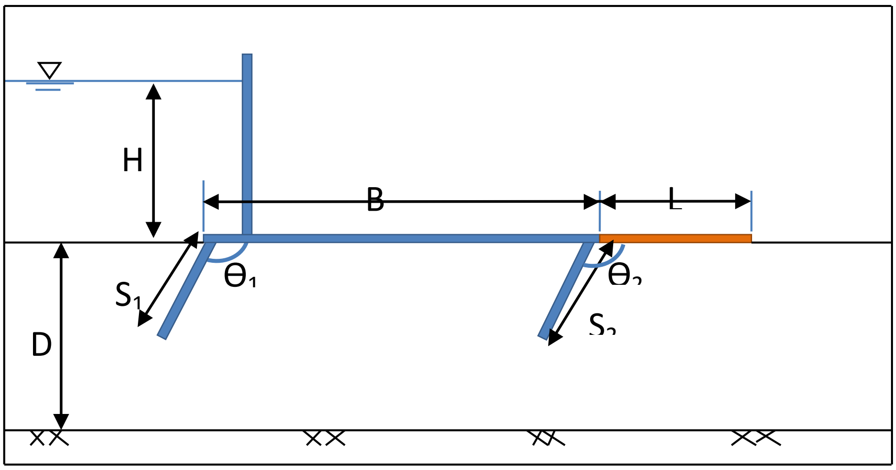

Figure 1. The variables involved in the problem.

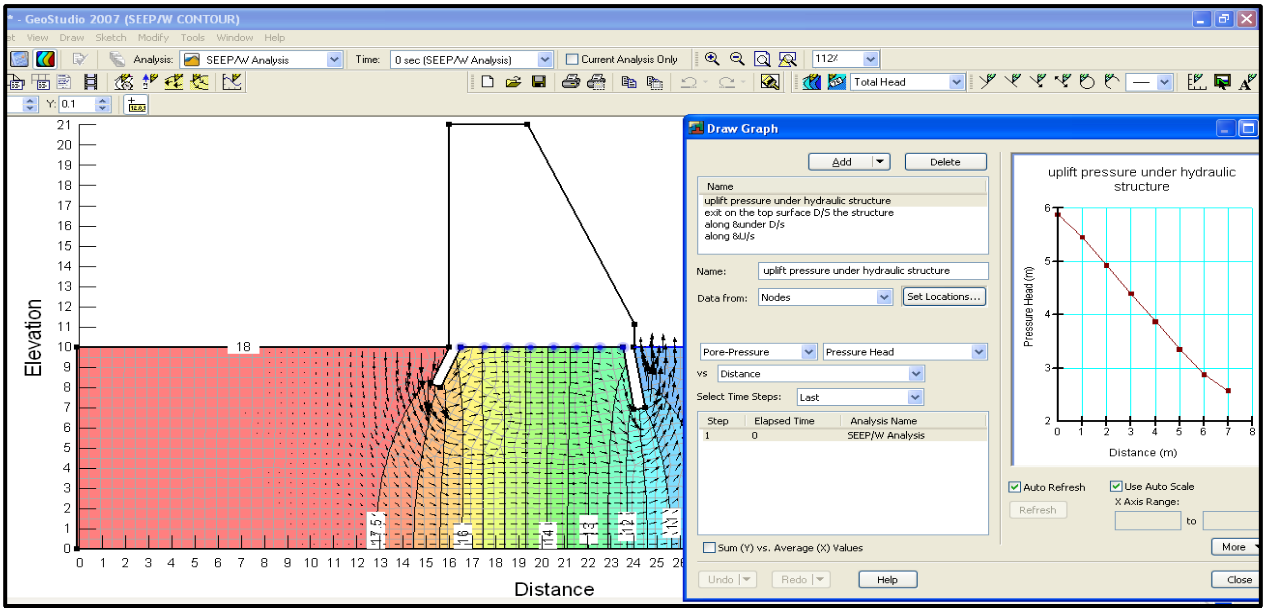

Figure 2. Uplift Pressure Distribution beneath the Structure, flow lines and equipotential lines.

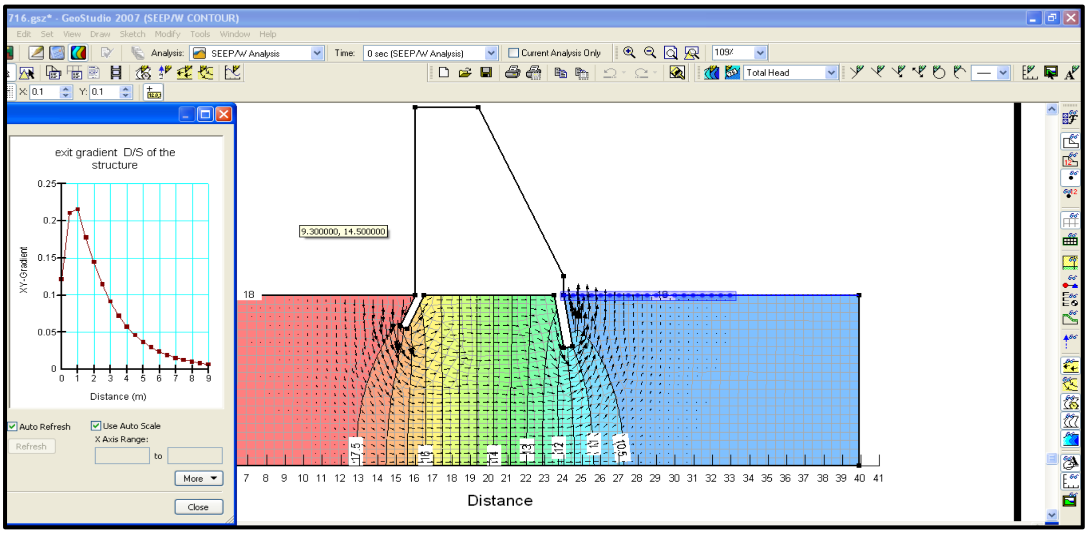

Figure 3. Distribution of the exit gradient along the downstream Side.

(The zero distance in the distribution figure refer to the right point of the dam base)

Table 1. Results obtained for L and V using the Geo-studio Models (k=4.889*10-4, k_x/k_y=2, H=6m, B=12m, D=10m).

S_1 (m)	S_2 (m)	θ_1	θ_2	L	V
1	1	120	80	1.0	140
1	2	115	80	0.0	152
1	3	110	80	0.0	164
1	4	105	80	0.0	174
2	1	95	80	0.9	133
2	2	90	80	0.0	145
2	3	85	80	0.0	158
2	4	80	80	0.0	169
3	1	70	80	0.9	144
3	2	65	80	0.0	158
3	3	60	80	0.0	174
3	4	50	80	0.0	192
4	1	120	80	0.7	117
4	2	115	80	0.0	128
4	3	110	80	0.0	139
4	4	105	80	0.0	151
2	2	80	110	0.7	143
2	3	75	110	0.5	154
2	4	70	110	0.4	167
3	1	60	110	0.8	136
3	2	50	110	0.7	150
3	3	130	110	0.4	137

Table 2. Data division percent.

Item		N	% Total output
Sample	Training	692	57.70%
	Testing	311	25.90%
	Validation	197	16.40%
Total		1200	100.00%
Excluded		0	

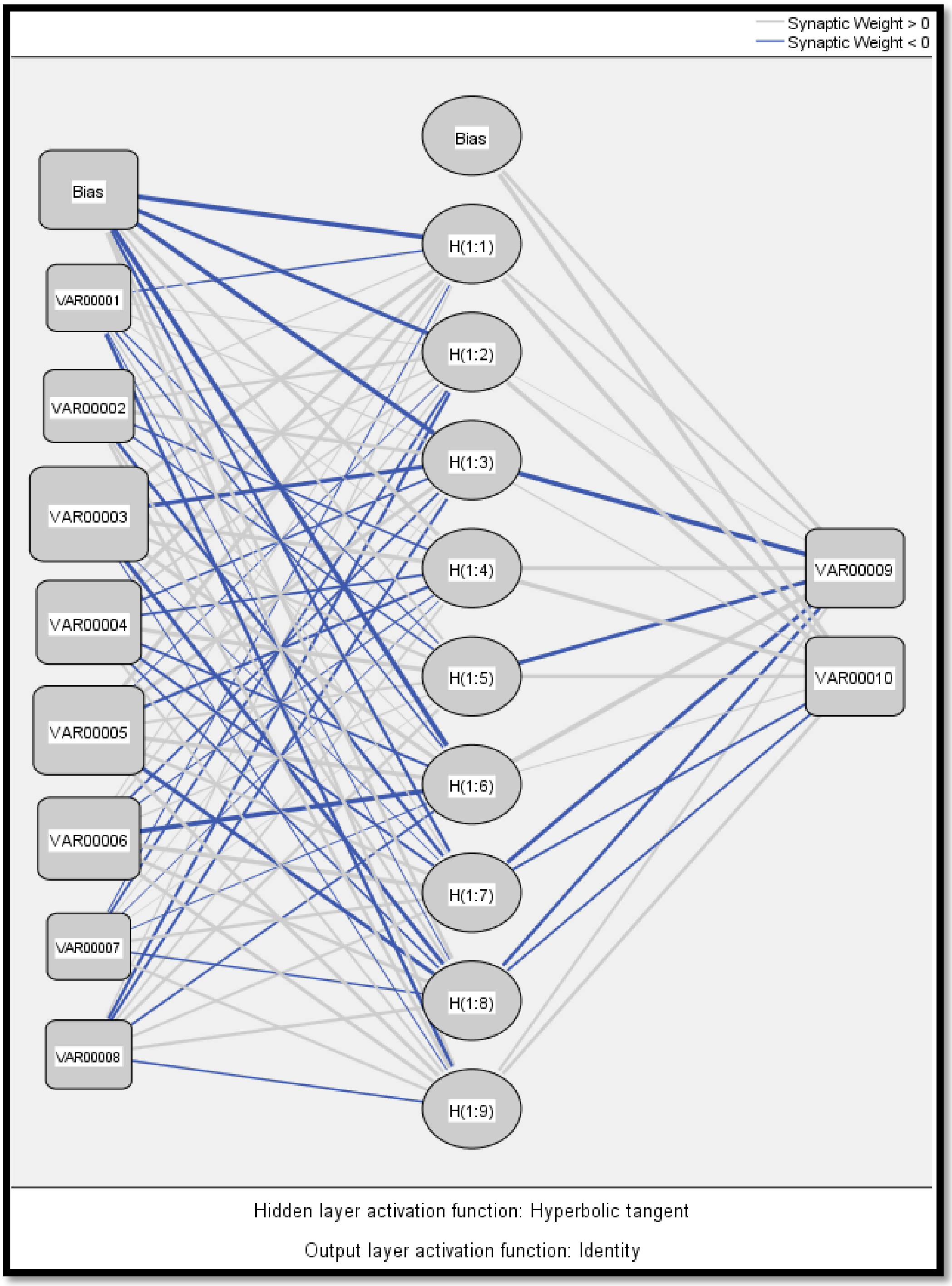

Figure 4. Architecture of the artificial neural network model.

Table 3. Bias and weight matrices

Parameter Estimates

Predictor		Predicted										
		Hidden Layer 1									Output Layer	
		H(1:1)	H(1:2)	H(1:3)	H(1:4)	H(1:5)	H(1:6)	H(1:7)	H(1:8)	H(1:9)	VAR00009	VAR00010
Input Layer	(Bias)	-1.506	-0.552	-1.345	0.596	0.17	-1.826	-0.233	0.358	0.277		
	VAR00001	-0.084	0.041	0.059	0.022	-0.052	-0.086	0.056	-0.032	-0.345		
	VAR00002	0.054	0.218	0.349	-0.102	-0.091	0.032	-0.261	0.195	0.325		
	VAR00003	0.632	0.1	-0.572	0.731	0.238	0.569	-0.009	-0.442	-0.037		
	VAR00004	0.58	0.124	-0.112	-0.137	1.022	-0.165	-0.187	-0.106	0.155		
	VAR00005	0.555	-0.023	0.427	-0.212	0.183	0.655	0.275	-0.434	0.421		
	VAR00006	0.037	0.064	-0.128	-0.046	0.062	-1.467	1.187	0.366	0.382		
	VAR00007	-0.045	-0.155	0.011	-0.022	0.007	-0.037	0.252	-0.095	0.208		
	VAR00008	0.062	-0.22	-0.193	0.124	0.246	-0.128	0.186	0.26	-0.104		
Hidden Layer 1	(Bias)										0.39	1.096
	H(1:1)										0.212	1.354
	H(1:2)										0.008	0.747
	H(1:3)										-1.801	0.055
	H(1:4)										0.492	0.795
	H(1:5)										-0.478	0.373
	H(1:6)										2.208	0.047
	H(1:7)										-0.432	-0.142
	H(1:8)										-0.269	-0.141
	H(1:9)										0.197	0.315

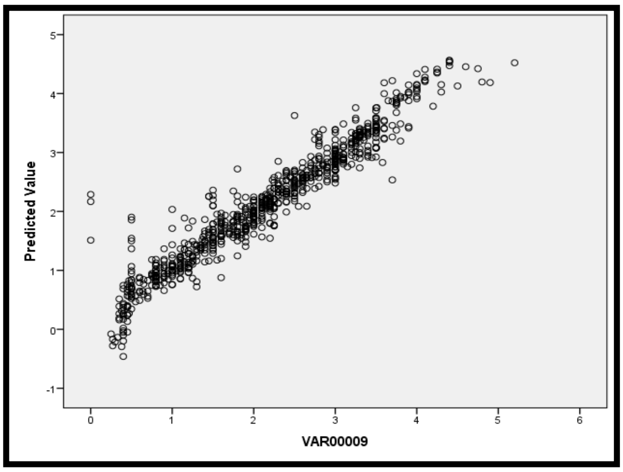

Figure 5. Comparison between predicted and observed values of (L).

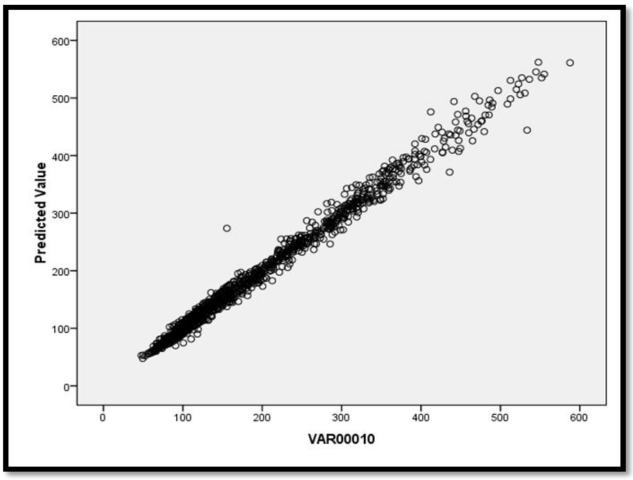

Figure 6. Comparison between predicted and observed values of (V).

Table 4. Comparison of (L and V) values using geo-studio and ANN model.

Case No.	S1 (m)	S2 (m)	H (m)	B (m)	D (m)	Kr	θ1	θ2	Geo-studio Model		ANN Model		% Difference for L Value	% Difference for V Value
									L	V	L	V		
1201	1.5	1.5	10.5	11	10	3	95	85	1.7	223.82	1.68	223.89	1.45	-0.03
1202	1.5	2.5	10.5	11	10	3	90	85	1.7	238.28	1.64	241.21	1.92	-1.23
1203	1.5	3.25	10.5	11	10	3	85	85	1.5	249.18	1.49	255.94	0.36	-2.71
1204	1.5	3.75	10.5	11	10	3	80	85	1.2	261.94	1.33	267.51	-11.2	-2.13
1205	2.5	1.5	9.25	11	10	1	70	105	2.3	194.84	2.14	191.91	6.78	1.506
1206	2.5	2.5	9.25	11	10	1	65	105	2.1	212.43	2	206.09	4.82	2.985
1207	2.5	3.25	9.25	11	10	1	60	105	1.9	227.01	1.81	220.56	4.84	2.842
1208	2.5	3.75	9.25	11	10	1	120	105	1.7	213.17	1.52	204.83	10.76	3.913
1209	3.5	1.5	7.5	11	10	2	115	115	0.8	135.02	0.71	138.64	11.14	-2.68
1210	3.5	2.5	7.5	11	10	2	110	115	0.7	145.49	0.66	148.96	5.66	-2.39
1211	3.5	3.25	7.5	11	10	2	105	115	0.6	153.66	0.5	158.98	8.44	-3.46
1212	3.5	3.75	7.5	11	10	2	95	115	0.4	161.12	0.34	168.86	8.11	-4.8
1213	3.75	1.5	6.25	11	10	4	90	80	0	120.67	0.17	117.53	-	2.602
1214	3.75	2.5	6.25	11	10	4	85	80	0	143.23	0.12	138.03	-	3.631
1215	3.75	3.25	6.25	11	10	4	80	80	0	160.68	0	156.86	-	2.376
1216	3.75	3.75	6.25	11	10	4	75	80	0	176.39	0	170.61	-	3.276

Table 5. Experimental result concerning the sensitivity analysis of the probability of crossover (pc) .(k=2, C_{S1}=0.1, C_{S2}=0.1, C_B=0.2, C_L=0.2 and C_V=0.4).

pc value	np value	Run 1	Run2	Run3	%Difference for Run1 & Run2	%Difference for Run1 & Run3	%Difference for Run2 & Run3
1	30000	65.5	66.58	65.99	-1.6489	-0.7481	0.8862
0.8	30000	65.73	65.84	66.38	-0.1674	-0.9889	-0.8202

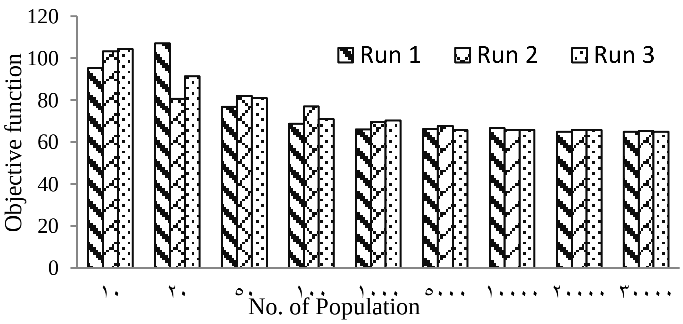

Figure 7. Variation of the objective function with respect of number of population initial randomly generated (k=4, C_{S1}=0.2, C_{S2}=0.2, C_B=0.1, C_L=0.1, C_V=0.4 and pc=1).

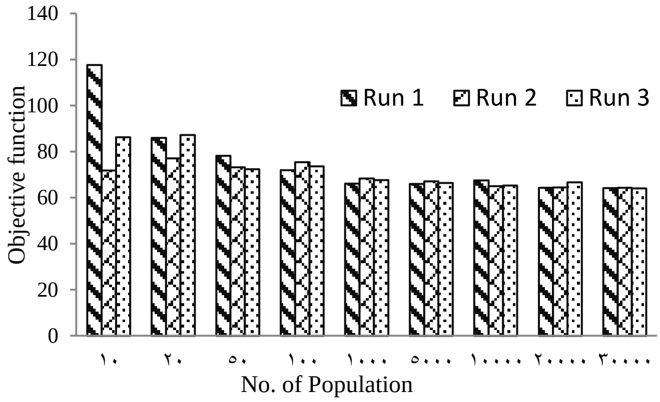

Figure 8. Variation of the objective function with respect of changing the integer position (k=2, C_{S1}=0.2, C_{S2}=0.2, C_B=0.1, C_L=0.1, C_V=0.4 and pc=1).

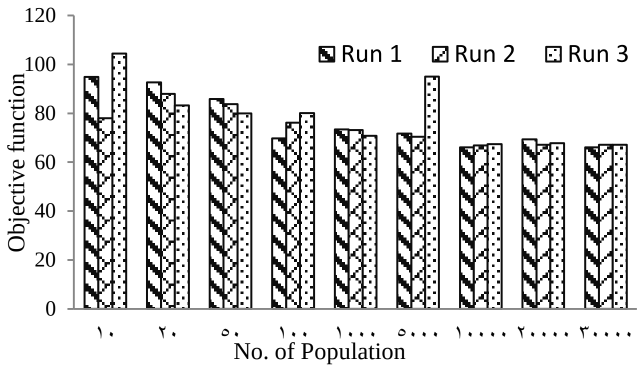

Figure 9. Variation of the objective function with respect of changing the integer position (k=7, C_{S1}=0.2, C_{S2}=0.2, C_B=0.1, C_L=0.1, C_V=0.4 and pc=1).

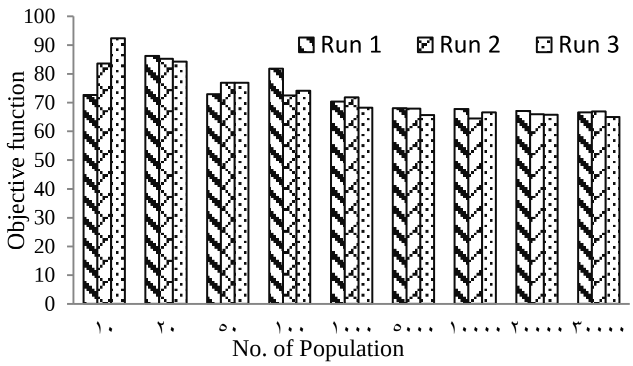

Figure 10. Variation of the objective function with respect of changing the integer position (k=1, C_{S1}=0.2, C_{S2}=0.2, C_B=0.1, C_L=0.1, C_V=0.4 and pc=1).

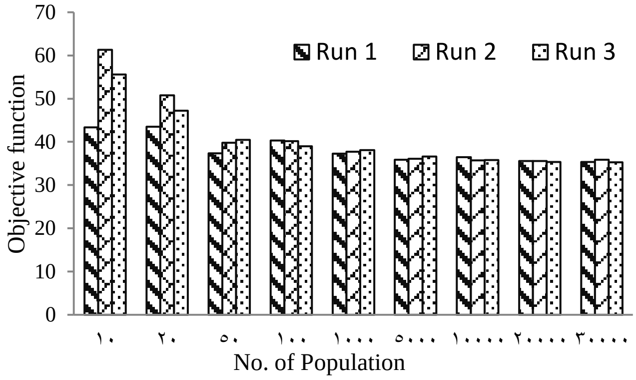

Figure 11. Variation of the objective function with respect of changing the relative weights of the objective function (k=2, C_{S1}=0.2, C_{S2}=0.2, C_B=0.2, C_L=0.2, C_V=0.2 and pc=1).

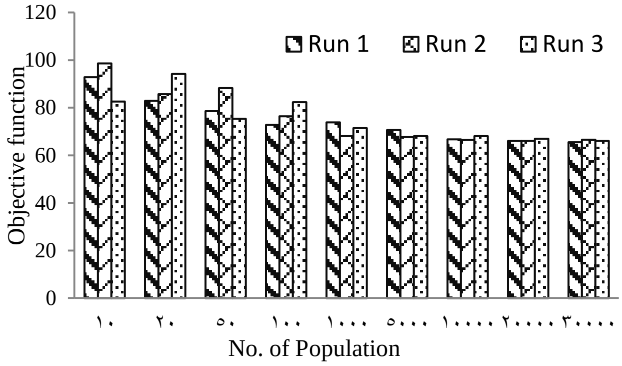

Figure 12. Variation of the objective function with respect of changing the relative weights of the objective function (k=2, C_{S1}=0.1, C_{S2}=0.1, C_B=0.1, C_L=0.3, C_V=0.4 and pc=1).

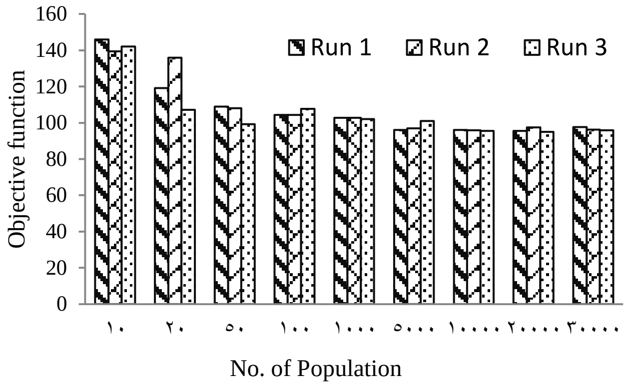

Figure 13. Variation of the objective function with respect of changing the relative weights of the objective function (k=2, C_{S1}=0.1, C_{S2}=0.1, C_B=0.1, C_L=0.1, C_V=0.6 and pc=1).

Table 6. Optimum solution obtained using GA model H=10m, Kr=1, D=10m, pc=1, k=2, C_{S1}=0.2, C_{S2}=0.2, C_B=0.2, C_L=0.2, C_V=0.2.

Run no.	S_1 (m)	S_2 (m)	B (m)	θ_1	θ_2	L (m)	V (m^3)	F(x)
1	3.9	0.53	10.3	128.5	51.2	3.613	159.49	35.59
2	3.97	0.61	10.1	129.3	68.1	3.24	159.4	35.4
3	3.94	0.57	10.1	119.6	64.3	3.44	160.6	35.9

Table 7. Effect of mutation on the value of the optimum solution.

	modified variable		F(x)	modified variable		F(x)	modified variable		F(x)
Run 1 (S_1=3.9, S_2=0.53, B=10.3, θ_1=128.5, θ_2=51.2, F(x)=35.59)	θ_1	118.5	36.5	θ_1	130	35.4	θ_1	110	37
	θ_2	61.1		θ_2	50.2		θ_2	51.2	
	S_1	3.8	35.7	S_1	3.7	36.1	S_1	4	35.1
	S_2	0.53		S_2	0.63		S_2	0.4	
	B	11.3	39.3	B	10	34.1	B	9.9	34
	S_1	4		S_1	4		S_1	3.8	
	B	9.9	34	B	10.1	34.9	B	10.5	36.4
	S_2	0.63		S_2	0.63		S_2	0.53	
	S_1	4	35.7	S_1	3.8	35.6	S_1	4	36
	θ_1	123.5		θ_1	130		θ_1	118.5	
	S_2	0.63	35.9	S_2	0.73	36	S_2	0.5	35.8
	θ_2	55.2		θ_2	53.2		θ_2	60.2	
	modified variable		F(x)	modified variable		F(x)	modified variable		F(x)
Run 2 (S_1=3.97, S_2=0.61, B=10.1, θ_1=129.3, θ_2=68.1, F(x)=35.4)	θ_1	125.3	35.5	θ_1	129.3	35.5	θ_1	125.3	35.7
	θ_2	68.1		θ_2	73.1		θ_2	73.1	
	S_1	3.8	35.69	S_1	4	35.5	S_1	3.9	35.8
	S_2	0.61		S_2	0.71		S_2	0.81	
	B	10.5	36.8	B	9.6	33.67	B	9.6	34.9
	S_1	4		S_1	3.8		S_1	4	
	B	10.5	36.9	B	9.6	33.5	B	9.6	33.6
	S_2	0.61		S_2	0.71		S_2	0.81	
	S_1	3.8	35.8	S_1	4	35.3	S_1	3.9	35.7
	θ_1	125.3		θ_1	129.3		θ_1	125.3	
	S_2	0.61	35.5	S_2	0.71	35.5	S_2	0.81	35.9
	θ_2	73.1		θ_2	68.1		θ_2	73.1	
	modified variable		F(x)	modified variable		F(x)	modified variable		F(x)
Run 3 (S_1=3.94, S_2=0.57, B=10.1, θ_1=119.6, θ_2=64.3, F(x)=35.9)	θ_1	114.6	35.8	θ_1	119.6	35.9	θ_1	114.6	36.1
	θ_2	46.3		θ_2	69.3		θ_2	69.3	
	S_1	3.8	35.7	S_1	4	35.95	S_1	3.8	35.2
	S_2	0.57		S_2	0.67		S_2	0.67	
	B	10.6	36.2	B	10.1	34.6	B	10.6	35.1
	S_1	4		S_1	3.8		S_1	3.8	
	B	10.6	34.3	B	10.1	35.7	B	10.6	34.8
	S_2	0.57		S_2	0.67		S_2	0.67	
	S_1	3.8	35.6	S_1	4	35.4	S_1	3.8	35.5
	θ_1	114.6		θ_1	119.6		θ_1	114.6	
	S_2	0.57	35.8	S_2	0.67	35.6	S_2	0.67	35.7
	θ_2	46.3		θ_2	69.3		θ_2	69.3	

Table 8. Comparison of estimated (L and V) values between Geo-studio model and GA model

Case No.	Given Values			Estimated Values										% Difference for (L) value	% Difference for (V) value
				Genetic Algorithm Model							Geo-studio Model			
	H (m)	D (m)	K_r	S_1 (m)	S_2 (m)	B (m)	θ_1	θ_2	L (m)	V (m^3)	L (m)	V (m^3)		
1	6	10	2	3.9	0.56	6	128	80	1.45	33.12	1.3	36.5	-11.5	9.26
2	8	10	4	4	0.51	8.1	127	59.5	1.18	72	1.1	78.9	-7.27	8.745
3	10	10	1	4	0.61	10	129	68.1	3.24	159.4	3.3	144	1.818	-10.69