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Non-deterministic Approach for Reliability Evaluation of Steel Beam

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ABSTRACT

This paper aims to evaluate the reliability analysis for steel beam which represented by the probability of Failure and reliability index. Monte Carlo Simulation Method (MCSM) and First Order Reliability Method (FORM) will be used to achieve this issue. These methods need two samples for each behavior that want to study; the first sample for resistance (carrying capacity R), and second for load effect (Q) which are parameters for a limit state function. Monte Carlo method has been adopted to generate these samples dependent on the randomness and uncertainties in variables. The variables that consider are beam cross-section dimensions, material property, beam length, yield stress, and applied loads. Matlab software has been adopted to generate these pseudo-random variables dependent on its statistical characteristics such as coefficient of variance and probability density function that gathered from a review of literatures.

Keywords: Reliability analysis, Monte Carlo Method, Matlab.

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الخلاصة

أن الهدف من دراسة هذه الورقة البحثية هو لتقييم تحليل الموثوقية لعتبة حديدية والمتمثلة بحساب احتمالية الفشل و معامل الموثوقية. تتم هذه المسالة باستخدام طريقتين الاولى تحليل الموثوقية بالدرجة الاولى و الثانية استخدام محاكاة مونتي كارلو لتحليل الموثوقية. عند استخدام هذه الطريقتين نحتاج لعينتين من تصرف العتبة المطلوب دراسته العينة الاولى لقابلية تحمل او الممانعة و العينة الثانية تصرفها نتيجة الحمل المسلط وهما يمثلان متغيرات لمعادلة دالة الحد. تم توضيف طريقة مونتي كارلو لتوليد هذه العينات بالاعتماد على العشوائية وعدم اليقين في المتغيرات. المتغيرات التي اخذت بنظر الاعتبار هي ابعاد المقطع

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العرضي للعتبة, خاصية المادة, طول العتبة, اجهاد الخضوع, و الاحمال المسلطة. يتم استخدام برنامج ماتلاب لانشاء المتغيرات العشوائية الكاذبة بالأعتماد على خصائصها الاحصائية من معامل التغايير و نوع توزيع الدالة التي تم جمعها من الادبيات السابقة.

الكلمات الرئيسية: تحليل الموثوقية, طريقة مونتي كارلو, برنامج ماتلاب.

1. INTRODUCTION

The design of engineering structures is usually associated with a significant level of uncertainties due to limited information in the process of estimating the structural parameters. The impact of uncertainties needs to be quantified and propagated to obtain the reliability of a structural system **(Morio & Balesdent, 2016).** In practice, most engineering design of structures are based on deterministic parameters and often do not consider the variations in the material properties and the geometry of the structure. **(Ebenuwa & Tee, 2019)** stated that the determination of structural performance based on the deterministic model is undoubtedly a simplification because physical measurement always shows variability and randomness.

In many circumstances, it is impossible to describe the response of structural systems mathematically because of these uncertainties. Even after finding a mathematical model to predict the behavior of the system, there is no closed form solution for solving the equation. In such cases, simulation is one of the most applicable techniques to acquire the required information. Simulation is a special technique to approximate the quantities that are difficult to obtain analytically. Amongst many of simulation methods, the Monte Carlo simulation method is one of the well-known and common procedures in solving complex engineering problems **(Melchers & Beck, 2017)**.

Theory and methods for structural reliability have been developed substantially in the last few years and they are actually a useful tool for evaluating rationally the safety of complex structures or structures with unusual designs **(Gordini, et al., 2018).** Recent evolution allows anticipating that their application will gradually increase, even in the case of common structures **(Cardoso, et al., 2008).**

The behavior of steel beam is generally assessed based on their strength and their elastic deformations In addition to the deterministic aspects that discussed in mechanics of material, the strength and deformation of steel beams have random parts due to the scatter in the dimensions, material properties, and the applied load. These random aspects can be simulated in terms of the probability density functions that either obtained from real experimental data on the member scale level or from the simulation that based on data of sectional level **(Ghali, et al., 2009).**

This paper starts with data gathering from literature for the variation in cross-section dimensions of frame elements, the variation in the elastic modulus and yield stress of the material, and the scatter in the applied loads. Based on these data, it has been found that the variation in the sectional dimensions, elastic modulus, yield stress, and dead loads are normally distributed while the lognormal and extreme type I (Gumbel) can be adopted for the variation in the length and live loads respectively.

Monte Carlo simulation has been used to generate a sample for the parameters that effected on the beam behavior. Two samples have been generated first one is the demand sample while the second one is the capacity samples. These samples had been presented and summarized in the form of histograms. The generated sample has been statistically tested with the χ^2 test. Base on limit state function, these samples have been used to estimate the probability of failure for a steel beam. This study innovatively concerns with the randomness in structural parameters and how these randomness effects on structure reliability by determining the probability of failure and reliability index.

2. UNCERTAINTY IN ENGINEERING SYSTEM

Every structure may contain some failed elements which lead to the whole system failure. The probability of failure for the system can be predicated established on the failure of its elements. Hence, it is significant in reliability analysis to determine the probability of system elements failure. First and second-order of reliability method and Monte Carlo methods can be used to analyze the reliability of elements **(Mohammad Masoud & Medi Moudi, 2012).** For the statically determinate simply supported beam of this paper, the element failure is equivalent to the system failure.

The uncertainties included in the building engineering can be categorized according to their source into natural hazards and man-made hazards. Natural hazards may be resulted by wind, seismic, temperature differentials, snow load, or ice accretion. The natural variations of structural properties such as strength, stiffness and loads can be classified within the natural hazards. On the other hand from the structural point of view, the man-made hazards can be subclassified into two classes: from within the building process and from outside the building process. The second one includes uncertainties due to fires, gas explosions, collisions, and similar causes, while the first one includes uncertainties due to acceptable practice and those caused by departures from acceptable practice (**Nowak & Collins, 2000**). This paper concerns with the natural hazard aspects due to change in stiffness, strength, and the applied loads of simply supported beam.

3. PROPOSED STRUCTURAL SECTIONS

In steel beam floor system, the members that are oriented parallel to the span of the slab system are usually referred to as beams, and the members that support the beams and are oriented perpendicular to the span of the slab system are usually called girders (**Al-Zaidee & Al-Hasany, 2018**).

This paper considers the reliability analysis of the interior girder for the floor system shown in **Fig. 1**. The floor system consists of a concrete slab with a corrugated metal deck that supported by four-floor beams that in turn are supported by three girders. The proposed sections for different members indicated in **Fig. 1** below has been preliminarily selected based on traditional design requirements (**AISC 360, 2010**). Uniformly distributed pressures of 2kPa and 2.87 kPa have been adopted for the superimposed and live loads respectively. According to the traditional one-way analysis, these loads are transformed into line loads supported by the floor beams. The reactions from the floor beams are applied as point loads on the supporting girders. For the interior girder, this analysis process leads to concentered forces of 118.76 kN and 59.4 kN for dead and live load reactions respectively. In subsequent simulation analysis, the live load reaction has been used as the mean value while the dead load reaction has been slightly modified to be considered as a mean value.
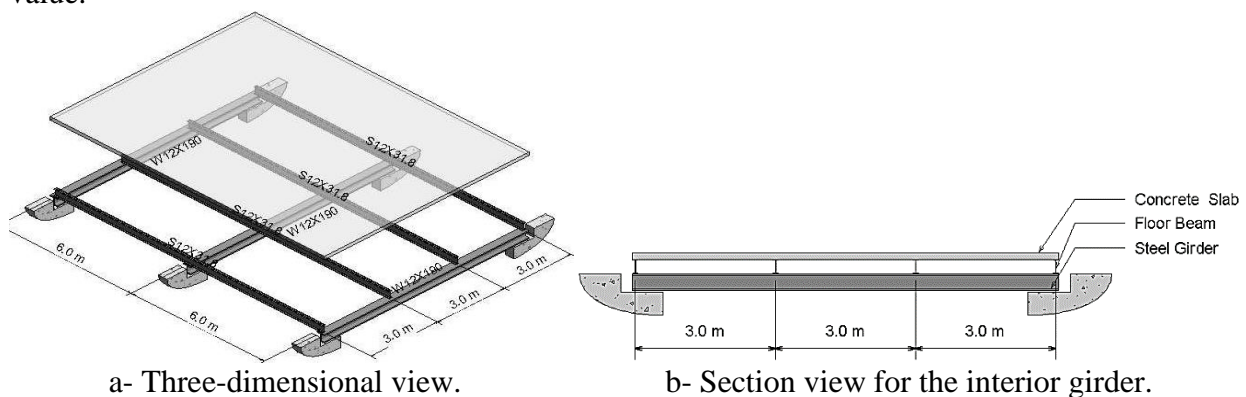

a- Three-dimensional view.   b- Section view for the interior girder.
Figure 1. Floor system 3D views.

4. LIMIT STATE FUNCTIONS (PERFORMANCE FUNCTIONS)

In this paper, the serviceability limit state deflection function and ultimate moment limit state function have been studied for the beam. Traditionally, when a beam is progressively loaded, the deflection linearly increased at an elastic stage **(Jabir, et al., 2017)** and the ultimate limit states can be used to determine the safety margin. Consider the moment carrying the strength of the beam to indicate the capacity, R, and the applied moment at the most critical mid-span section to indicate the demand, Q, the performance function can be written as follows:

$$g(R.Q) = R - Q \tag{1}$$

The beam is classified safe when $g \geq 0$ while it is unsafe when $g < 0$. Mathematically, the failure probability P_f is equal to the probability of $g < 0$:
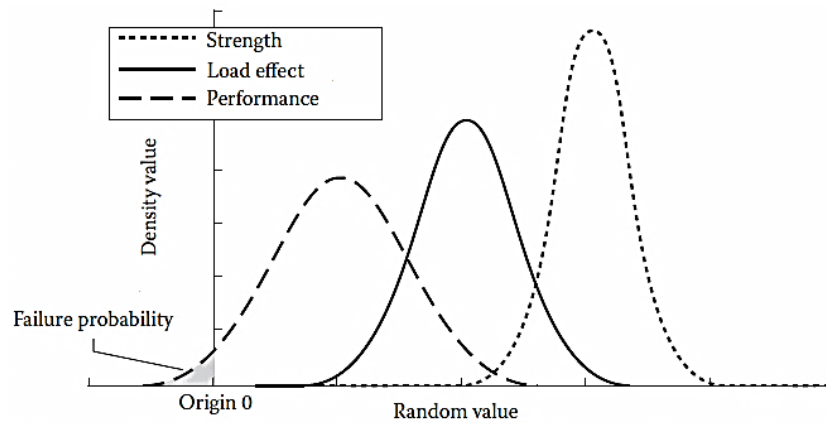
$$P_f = P(g < 0) = P(R - Q < 0) \tag{2}$$

Figure 2. PDFs of load, resistance, and safety margin (Ayyub & McCuen, 2011).

If R and Q have probability density functions (PDF) indicated in **Fig. 2**, the quantity R-Q would be a random variable also with its own PDF. As shown in **Fig. 2**, the probability of failure would correspond to the shaded area.

In general, the performance function, g, may be a function of many variables including loads, influence factors, strength parameters, material properties, dimensions, analysis factors, and so on. A direct determinate of P_f from Eq. (2) is relatively difficult. Therefore, it would be more appropriate to express structural safety in the expression of a reliability index, β, which can be described as the shortest distance from the origin to the failure limit. When R and Q are uncorrelated the reliability index, β, would be the inverse of the coefficient of variation of the Eq. (1) **(Nowak & Collins, 2000)**:

$$\beta = -\varphi^{-1}(P_f) \; or \; P_f = \varphi(-\beta) \tag{3}$$

Theory and methods for structural reliability that have been originally developed as a useful facility for determining rationally the safety of complicated and unusual structures or structures with unusual designs **(Cardoso, et al., 2008).**

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From a statistical point of view, the PDF of $g(R,Q)$ and β can be determined either analytical or based on a simulation process. Monte Carlo technique has been used for a simulation to determine the reliability index β numerically see Section 6. The analytical determination of β has been presented in Section 7.

In this study, the reliability analysis for beam has been studied with two scenarios, first by considering only the applied load as constant by using their mean values and other parameters as variables due to randomness, the second scenario by considering loads as variables and other parameters as constant using their mean values.

5. RANDOM VARIABLES WITH THEIR STATISTICAL PARAMETERS

5.1   Geometric Characteristics of Hot-Rolled Profiles

In their work (**Zdenek Kala, et al., 2009**) gathered 369 valid observations for the variables $h, b_1, b_2, t_1, t_{21}, t_{22}$, indicated in **Fig. 3** from a manufacturer and analyzed the data statistically to evaluate the suitability of the normal distribution as a governing distribution for these dimensions. As indicated in **Table 1**, they presented the relative (non-dimensional) geometrical characteristic as ratios of the real measured to the corresponding nominal dimension.

a- Geometric characteristics $\boldsymbol{h}, \boldsymbol{b}, \boldsymbol{t_1}, \boldsymbol{t_{21}}, \boldsymbol{t_{22}}$.   b- Geometric characteristics $\boldsymbol{b_1}, \boldsymbol{b_2}$.

Figure 3: Tolerances on geometrical shape and dimensions.

As indicated in **Table 1**. (**Zdenek Kala, et al., 2009**) have noted that for a symmetrical cross-section the statistical characteristics of quantities t_{21} and t_{22} are approximately identical and that there is a small difference between statistical characteristics of the quantities b_1 and b_2. Therefore, they adopted a single random variable of t_2 for each of t_{21} and t_{22} and a random variable of b for b_1 and b_2 in the reliability analysis.

Table 1. Statistical analysis of geometric characteristics.

Thickness	Mean value	Standard deviation
Section depth h	1.0009	0.0044233
Section width b_1	1.0124	0.010103
Section width b_2	1.0154	0.0093995
Section width b	1.0139	0.009868
Web thick. t_1	1.0540	0.039053
Flange thick. t_{21}	0.9878	0.043528
Flange thick. t_{22}	0.9977	0.047625
Flange thick. t_2	0.9927	0.045859

Depending on the nominal dimensions of wide flange steel sections and the non-dimensional variations indicated **Table 1** above, randomness for the moment of inertia have been simulated in

this paper using a Matlab code and suitable random number generators. The four-moment statistical characteristics of the mean, the variance, the coefficient of skewness, and the coefficient of kurtosis have been determined and a normal distribution probability density function, pdf, with parameters indicated in **Table 2** has been assumed for the generated data for the moment of inertia. Adequacy of the proposed pdf has been checked using the χ^2 goodness of fit test.

Table 2. Statistical characteristic for the moment of inertia.

Property	Mean/nominal	Cov	Distribution type
I	1.0	0.035	Normal

5.2   Applied Loads

In addition to its own weight, W_D, the interior girder is subjected to two concentrated loads F_D, and F_L transformed from the supported floor beams. According. (**S.G.Buonopane & B.W.Schafer, 2006**), the dead load has a normally distributed pdf while the live load follows an extreme type I (Gumbel) distribution with statistical characteristics illustrated in **Table 3.**

5.3   Yield Stress and Residual Stresses

Due to the effects of the residual stresses, the yield stresses will vary through the section of hot rolled steel beams. According to (**J. Kala & Z. Kala, 2005**), this variation can be described based on parameters and statistical distributions indicated in **Table 4**. Based on these data, a Matlab random number generator has been used in this paper to generate a sample of yield stresses values that have been used in subsequent calculations of the nominal flexural strength, M_n, of the interior girder.

Table 3. Statistical characteristic for loads.

Random variables	Nominal load	Mean	COV	Standard deviation	Distribution type	References
$W_D\ kN/m$	2.827	1.03 W_D =2.912	0.08	0.233	Normal	(M.Sigit Darmawan, et al., 2013)
$F_D\ kN$	118.76	1.03F_D =122.323	0.08	9.786	Normal	(M.Sigit Darmawan, et al., 2013)
$F_L\ kN$	59.4	59.4	0.1	5.94	Gumbel	(S.G.Buonopane & B.W.Schafer, 2006)

Table 4. Statistical characteristics of the yield stress of steel (J. Kala & Z. Kala, 2005)

No	Quantity	Name of a random quantity	Type of distribution	Dimensions	Mean value	Standard deviation
1	F_y	Flange yield strength	Normal (Gauss)	MPa	297.30	16.80
2	F_y	Web yield strength	Normal (Gauss)	MPa	307.30	16.80

5.4   Modulus of Elasticity and Length of the Element

Based on (**S.Zhang & W.Zhou, 2012**), and (**Mohammad Masoud & Medi Moudi, 2012**) parameters and statistical distributions indicated in **Table 5** has been used in this paper to simulate a sample data for the elastic modulus and beam length. Matlab random generators with the

corresponding distributions have been used to generate sample data for subsequent deflection and strength analysis.

Table 5. Statistical features of random variables.

Random variables	Mean/ Nominal	COV	Distribution type
Modulus of Elasticity E MPa	0.993	0.034	Normal
Length m	1	0.07	Lognormal

6. MONTE CARLO METHOD TO GENERATE SAMPLES

In this paper, all simulation processes for strength, serviceability, and reliability analyses have been achieved through the Monte Carlo method that has used digital computers to generate pseudo-random sampling for variables of dimensions, loads, elastic modulus, yield stress, and the girder span. Each variable has been generated based on preselected statistical parameters and distribution as discussed in Section 5. This section presents the application of the method for the simulation of strength and serviceability. The reliability aspects have been discussed in Section 7.

The method is based on running the model many times as in random sampling. For each sample, random variates are generated on each input variable; computations are run through the model yielding random outcomes on each output variable. Since each input is random, the outcomes are random **(Geng & Dean, 2017).** The method may be described as a means of solving problems numerically in mathematics, physics, and other sciences through sampling experiments **(Morio & Balesdent, 2016).**

In each simulation experiment, the possible values of the input random variables $x = (x_1 . x_2 . .... x_n)$ are generated based on predefined distribution and parameters. Then the values of the response variable, y, are determined through the performance function $y = g(x)$ at the samples of input random variables. In this manner, a set of samples for the response variable y would be available for the subsequent statistical analyses to estimate the characteristics of the response variable y (**Thomopoulos, 2013**).

The problem to be simulated may have a probabilistic or deterministic form. In the probabilistic form, the actual random variable or function appearing in the problem is simulated, whereas in the deterministic form an artificial random variable or function is first constructed and then simulated **(Elishakoff, 2017).** The interior girder of this paper can be classified as a deterministic form problem where the stiffness, strength, and stability response functions have been determined from the strength of the material and the design of steel structures.

For subsequent reliability analysis, the Monte Carlo method is used to generate samples for the resistance and the demand of the interior girder.

6.1   Analysis of Demand

As it is a statically determinate structure, the traditional equations Eq. (4) and Eq. (5), are used to calculate the deflection and moment at the mid-span of the girder as indicators on the demand aspects of serviceability limit state.

$$\Delta_t = \frac{5WL^4}{384EI} + \sum_{i=1}^{2} \frac{Fa^2b^2}{3EIL} \tag{4}$$

$$M = \frac{Wl^2}{8} + F_{(L+D)}\, a \tag{5}$$

Matlab codes have been used to generate pseudo-random numbers based on the following functions (**Ang & Tang, 2007**):

- *normrnd*: to generate normal random variables for the moment of inertia, the modulus of elasticity, the yield stress, and the dead load.
- *lognrnd*: to generate lognormal random variables for the beam span.
- *evrnd*: to generate extreme type I random variables for the live load.

As mention earlier, there are two scenarios to generate the demand sample. The first scenario considers the uncertainties in dimension, length, and modulus of elasticity as indicated in Matlab codes illustrated in **Table A-1** for deflection and **Table A-2** in Appendix A for the moment. The second scenario considered the uncertainties in loads and their position using Matlab codes presented in **Table A-3** and **Table A-4** for deflection and moment respectively. A samples size, N, of 10000 has been adopted in all Matlab codes.

The statistical properties for the obtained samples from the simulation process have been presented and discussed below:

- For the first scenario, the coefficient of variance, *standard deviation/mean*, for deflection and moment was equal to 0.071 and 0.007 respectively and each of them has a lognormal probability density function as illustrated in **Fig.4** and **Fig.5**. These results indicate that the deflection is more sensitive than the moment for the randomness in dimensions and material properties.
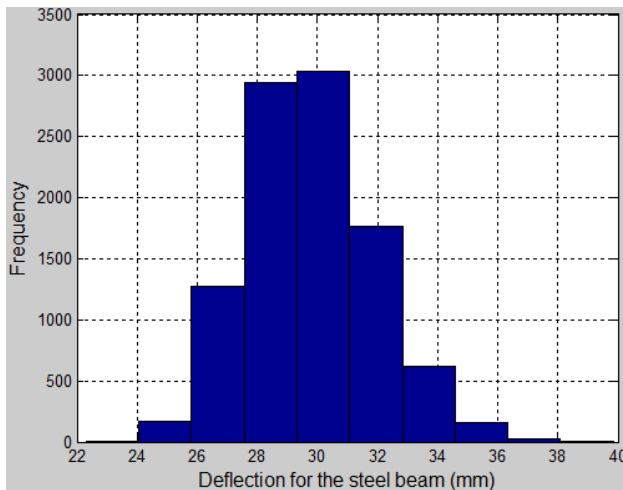
a- Histogram for deflection of the girder.   b- Moment characteristics.

Figure 4. Histogram and the statistical characteristics for the mid-span deflection due to randomness in dimensions, length, and material property.

```
M_mom =
    574.7891

v_mom =
    16.8202

S_mom =
    4.1014

B1_mom =
    0.4071

B2_mom =
    3.3148
```
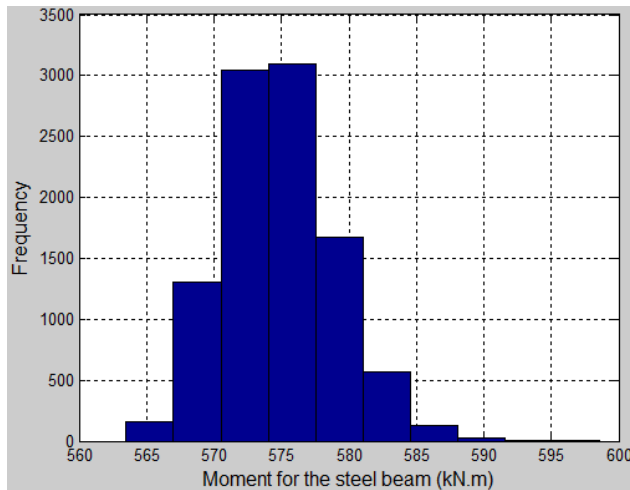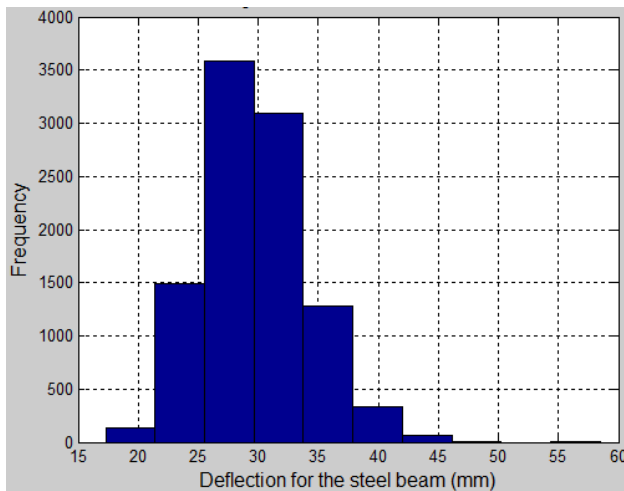
a- Histogram for the moment of the girder.   b- Moment characteristics

Figure 5. Histogram and the statistical characteristics for the mid-span moment due to randomness in dimensions, length, and material property.

- For the second scenario, the coefficients of variance were equal to 0.145 and 0.09 for the deflection and the moment respectively with lognormal distributions type as shown in **Fig. 6** and **Fig. 7**. The deflection and the moment seem more sensitive to the randomness in load than the randomness in the dimensions and material properties of the first scenario. In the two scenarios, the deflection is more sensitive to the randomness of the input variables.


```
M =
    29.7352

v =
    18.7060

S =
    4.3253

B1 =
    0.4653

B2 =
    3.4641
```
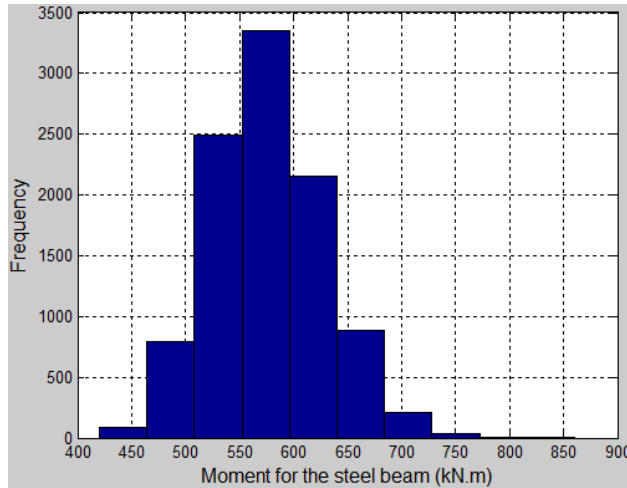
a- Histogram for deflection of the girder.   b- Moment characteristics.

Figure 6. Histogram and moment characteristics for deflection data due to randomness in self-weight, applied load, and their position.

a- Histogram for deflection of the girder.	b- Moment characteristics.

Figure 7. Histogram and moment characteristics for moment data due to randomness in self-weight, applied load, and their position.

6.2   Analysis of Resistance

The sample data for the deflection capacity and moment resistance have been simulated using Matlab functions similar to those mentioned in Section 6.1. For deflection, the capacity is represented by the maximum allowable deflection, while for the moment, the capacity is represented by elastic moment based on the assumption that the girder has no sufficient lateral support. The statistical characteristics for the resistance samples are presented and discussed below:

- For maximum allowable deflection:
  The random sample for the limit state deflection has been generated based on different random values for the girder span, ℓ, and the traditional ratio of $\ell/240$ for the permissible deflection due to dead and live. A mean to $\overline{\ell}/240$ has been adopted in this simulation process.
 Random sample for the girder span has been generated, plotted in a histogram form, and statistically analyzed using the Matlab code indicated in **Table A-5**. The histogram and the statistical characteristics values presented in **Fig.8** show that the generated sample has a lognormal distribution with a coefficient of variance equal to 0.069.
- For Moment capacity:
  The nominal moment capacity, M_n, can be estimated based on the elastic capacity, elasto-plastic capacity, or the full plastic capacity depends on the lateral support conditions and the compactness of the section. In this paper, the elastic moment indicated in Eq.(6) has been adopted based on insufficient lateral support.

$$M_n = M_y = F_y \times S_x \tag{6}$$

A Matlab code indicated in **Table A-6** has been prepared to generate a random sample for M_n based on the randomness of the yield stress, F_y, and the elastic section modulus, S_x. Histogram and the statistical characteristics for the obtained data have been presented in **Fig.9**. The generated random sample has a coefficient of variance of 0.053 with lognormal probability density function.
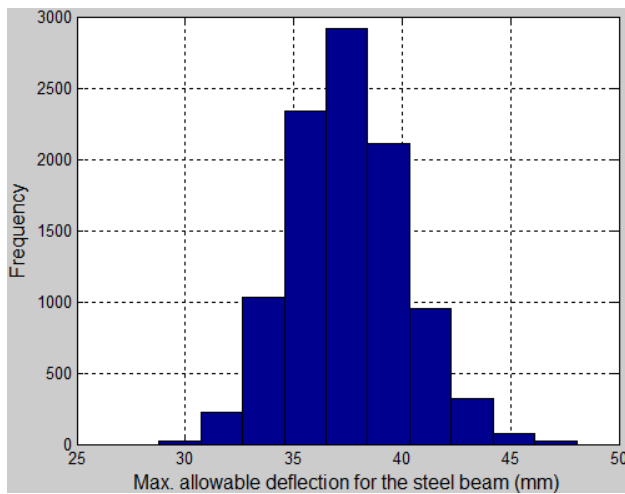
a- Histogram for Max. Allowable deflection.	b- Moment characteristics.

M_y =
　37.5029

v_y =
　6.7919

S_y =
　2.6063

B1_y =
　0.2500

B2_y =
　3.1512

Figure 8. Histogram and moment characteristics for maximum allowable deflection for the case of live and dead load.
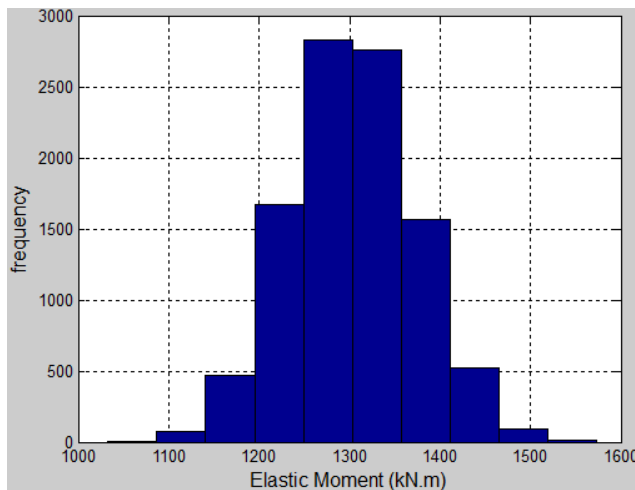
a- Histogram for resistance elastic moment.	b- Moment characteristics.

M_My =
　1.3032e+03

v_My =
　4.8070e+03

S_My =
　69.3359

B1_My =
　0.0881

B2_My =
　2.9527

Figure 9. Histogram and moment characteristics for the resistance of elastic behavior sample.

7. RELIABILITY ANALYSIS FOR BEAM

In this paper, the reliability analysis for the interior girder has been achieved using the Monte Carlo simulation method and the first order reliability method, FORM. These two methods have been discussed briefly in subsections below while their results and conclusions have been presented in Section 8.

7.1   Using Monte Carlo Simulation Method

In addition, to use Monte Carlo to generate random samples for demand and capacity, it provides a powerful approach for an approximation but an adequate simulation of the failure probability for N randomly generated samples based on the following relation (**Nowak & Collins, 2000**):

$$P_f = \frac{Number\ of\ trials\ for\ g(x) \leq 0}{N} \tag{7}$$

Accuracy of the estimated the probability increases as the total number of simulations, N, increases (**Far & Wang, 2016**).

7.2   First Order Second Moment Reliability Index

The first order reliability method, FORM, is another method to do reliability analysis for structure dependent on the statistical properties of resistance and demand samples for the limit state that want to study. It calculates the reliability index β then from Eq. (3) the P_f can be determined.

There are two cases to determine β dependent on limit state function if it is linear or nonlinear. For linear LSF when g expressed by:

$$g(X_1, X_2, \ldots, X_n) = a_0 + a_1 X_1 + a_2 X_2 + \cdots + a_n X_n = a_0 + \sum_{i=1}^{n}(a_i X_i) \tag{8}$$

where the a_i terms ($i = 0,1,2,\ldots,n$) are constants and the X_i terms are uncorrelated random variables. When R and Q are independent normally distributed random variables, (**AISC 360, 2010**), the reliability index computed as below:

$$\beta = \frac{\bar{R} - \bar{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{9}$$

where $\bar{R}$ and $\bar{Q}$ are mean values of R and Q respectively, σ_R^2 and σ_Q^2 are their variance values (Ghali, et al., 2009). If the independent random variables R and Q have lognormal random variable, β given as:

$$\beta = \frac{\mu L n\left(\frac{R}{Q}\right)}{\sqrt{V_R^2 + V_Q^2}} = \frac{Ln\mu_R - Ln\mu_Q}{\sqrt{V_R^2 + V_Q^2}} \tag{10}$$

where μ_R and μ_Q are mean values, V_R^2 and V_Q^2 are coefficients of variation of R and Q (**Popov, 1990**).

Observe that the reliability index depends only on the means and standard deviations of the random variables. Therefore, this β is called a second-moment measure of structural safety because only the first two moments (mean and variance) are required to calculate β (**Ghali, et al., 2009**).

For nonlinear LSF, an approximate answer can be obtained by linearizing the nonlinear function using two terms of a Taylor series expansion. The result is:

$$g(X_1, X_2, \ldots., X_n) \approx \bar{g} + \sum_{i=1}^{n} \left((X_i - \bar{x}_i) \ \frac{\partial g}{\partial X_i} \right) \tag{11}$$

where $\bar{g}$ is a value of g calculated with chosen values of the variables. One choice is the mean values of the random variables, giving an approximate mean value of g:

$$\bar{g} = g((\overline{x_1}, \overline{x_2}, \ldots., \overline{x_n}) = g(\{\bar{x}\}) \tag{12}$$

The first term in Eq. (11) is a constant; the remaining terms are linear combinations of the variables $(X_i - \bar{x}_i)$, with $\bar{x}_L$ constant, the approximate reliability index (**Ghali, et al., 2009**).

$$\beta \approx \frac{\overline{g}}{(\sum_{i=1}^{n}(a_i X_i)^2)^{\frac{1}{2}}} \ with \ a_i = \frac{\partial g}{\partial X_i}\bigg|_{at\ (\bar{x})} \tag{13}$$

The reliability index defined in Eq. (13) is called a first-order second-moment mean value reliability index. It is a long name, but the underlying meaning of each part of the name is very important: First order because of the use of first-order terms in the Taylor series expansion. Second moment because only means and variances are needed. Mean value because the Taylor series expansion is about the mean values (**Nowak & Collins, 2000**).

In this paper the LSF for each of deflection and moment are linear and all of resistance and demand samples have lognormal probability density function as illustrated in Section 6 therefor use Eq. (10) to calculate β.

8. RESULTS AND CONCLUSIONS

Based on statistical features of random variables mention earlier, failure probabilities for the beam are summarized as follows. When using the Monte Carlo simulation method the P_f for deflection due to randomness in dimensions and the material property is equal to 0.0114 and due to randomness in loads is equal to 0.0688. While for the strength limit state there are no trials for $g(x) \leq 0$ when variation due to dimension, material property, and due to applied loads, therefore, P_f for moment limit state function is negligible for these two scenarios.

By using the FORM and calculate β is equal to:

- β For deflection due to randomness in dimensions and material is equal to 2.333 and the corresponding P_f equal to 0.0098.
- β For deflection due to randomness in loads is equal to 1.445 and the corresponding P_f equal to 0.074.
- β For moment limit state function due to randomness in dimensions and material is equal to 15.312 and the corresponding P_f equal to 3.179×10^{-53}.
- β For moment limit state function due to randomness in loads is equal to 7.823 and the corresponding P_f equal to 2.579×10^{-15}.

It can notice that the results from two methods are very close and the P_f for the modes deal with variation in loads is greater than modes deal with variation in dimensions and material property. The deflection and the moment limit state functions seem more sensitive to the randomness in load than the randomness in the dimensions and material properties, and the deflection is more critical

to the randomness of the input variables. As a conclusion when failure probability is low, one can use element critical failure probability.

NOMENCLATURE

a, b = distance from beam supports to concentrated load, m.
B1 = coefficient of skewness.
B1 = coefficient of kurtosis.
E = modulus of elasticity, MPa.
F_D= concentrated superimposed load, kN.
F_L= concentrated live load, kN.
F_y= yield stress, MPa.
M= sample mean.
N = sample size.
Q = load effect (demand).
R = resistance (capacity).
$\bar{Q}$ = mean of Q
$\bar{R}$ = mean of R
S = standard deviation.
S_x= elastic section modulus.
v = variance.
W_D= uniform beam weight.
β = reliability index.
ℓ = span girder.
σ_R^2= variance R.
σ_Q^2= variance Q.
V_R^2 = coefficient of variance for R.
V_Q^2 = coefficient of variance for Q.
φ and φ^{-1}= standard normal cumulative distribution function and its invers.
FORM = first-order reliability method.
LSF = limit state function
MCSM = Monte Carlo simulation method.
P_f = probability failure.
PDF = probability density function.

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Appendix A Matlab Codes

Table A-1. Matlab code to generate a random sample, plot histogram, and determine the statistical properties for the mid-span deflection due to the first scenario.

```
1 -   clc
2     % Plot the Histogram for the Deflection of Steel Girder and Determine the
3     % Four Moments for the Data Obtianed Due to Variance in Dimension and Material Property
4     % Uniform Self Weight = WD
5     % Concentrated Live Load = FL
6     % Concentrated superimposed Load = FD
7     % Modulus of Elastisity = E
8     % Moment of Inertia = I
9     % Length of the Beam = L
10    % Distance from the support to the first point load at left = a1
11    % Distance from the support to the first point load at right = b1
12    % Distance from the support to the second point load at left = a2
13    % Distance from the support to the second point load at right = b2
14 -  N=10000  % Sample size in Monte Carlo experiment for deflection results.
15 -  y=zeros(1,N) % Initialize a vector to store deflection of the beam "y"
16 -  for i=1:N
17         % Normal model to simulate random variance in moment of inertia = I
18 -       I=normrnd(7.867*10^(-4),2.753*10^(-5))
19         % Normal model to simulate random variance in modulus of Elastisity = E
20 -       E=normrnd(1986*10^(5),6752400)
21         % Log normal model to simulate random variance in the length of beam = L
22 -       m = 9 % mean of beam length random values in m
23 -   v = 0.3969 % variance of beam length random values
24 -   mu = log((m^2)/sqrt(v+m^2))% mean for the log normal distribution
25 -   sigma = sqrt(log(v/(m^2)+1)) % standard deiviation for the log normal distribution
26 -   [M,V]= lognstat(mu,sigma)
27 -   L= lognrnd(mu,sigma)
28    % Distance from supports to applied loads in m
29 -   a1=3
30 -   b1=6
31 -   a2=3
32 -   b2=6
33    % Mean of uniform self weight WD in kN/m
34 -   WD=2.912
35    % Mean of concentrated superimposed load FD in kN
36 -   FD=122.323
37    % Mean of concenterated live load FL in kN
38 -   FL=59.4
39    % deflection due to uniform load = y1
40 -   y1=((5*(WD)*(L^4))/(384*E*I))
41    % deflection due to first point load = y2
42 -   y2=(((FD+FL)*(a1^2)*(b1^2))/(3*E*I*L))
43    % deflection due to second point load = y3
44 -   y3=(((FD+FL)*(a2^2)*(b2^2))/(3*E*I*L))
45    % the total deflection = y
46 -   y(1,i)=((y1+y2+y3)*1000)
47 -   end
48 -   xlswrite('y_dim.xlsx',y)
49 -   figure(1)
50 -   hist(y)
51 -   xlabel('Deflection for the steel beam (mm)','fontsize',12)
52 -   ylabel('Frequency','fontsize',12)
53 -   title('Histogram for Deflection of Steel Girder.')
54 -   grid
55 -   Max_y=max(y)
56 -   Min_y=min(y)
57 -   M=mean(y)
58    % Variance of y = v
59 -   v=var(y,1)
60    % standard deviation value of y = S
61 -   S=std(y)
62    % Coefficient of Skewness of y = B1
63 -   B1=skewness(y)
64    % Coefficient of Kutosis of y = B2
65 -   B2=kurtosis(y)
```

Table A-2. Matlab code to generate a random sample, plot histogram, and determine the statistical properties for the applied moment of the girder due to the first scenario.

```
1 -   clc
2     % Plot the Histogram for the Moment of Steel Girder and Determine the
3     % Four Moments for the Data Obtianed Due to Variance in Dimension and Material Property
4     % Uniform Self Weight = WD
5     % Concentrated Live Load = FL
6     % Concentrated superimposed Load = FD
7     % Modulus of Elastisity = E
8     % Moment of Inertia = I
9     % Length of the Beam = L
10    % Distance from the support to the first point load at left = a1
11    % Distance from the support to the first point load at right = b1
12    % Distance from the support to the second point load at left = a2
13    % Distance from the support to the second point load at right = b2
14 -  n=10000  % Sample size in Monte Carlo experiment for moment results.
15 -  mom=zeros(1,n) % Initialize a vector to store moment of the beam "mom"
16 -  for i=1:n
17      % Log normal model to simulate random variance in the length of beam = L
18 -      m = 9 % mean of beam length random values in m
19 -    v = 0.3969 % variance of beam length random values
20 -    mu = log((m^2)/sqrt(v+m^2))% mean for the log normal distribution
21 -    sigma = sqrt(log(v/(m^2)+1)) % standard deiviation for the log normal distribution
22 -    [M,V]= lognstat(mu,sigma)
23 -    L= lognrnd(mu,sigma)
24      % Distance from supports to applied loads in m
25 -    a1=3
26 -    b1=6
27 -    a2=3
28 -    b2=6
29      % Mean of uniform self weight WD in kN/m
30 -    WD=2.912
31      % Mean of concentrated superimposed load FD in kN
32 -    FD=122.323
33      % Mean of concenterated live load FL in kN
34 -    FL=59.4
35      % Moment in the middle of the girder due to uniform and concenterated load
36 -    mom(1,i)=(((WD*L^2)/8)+((FL+FD)*a1))
37 -  end
38 -  xlswrite('mom_dim.xlsx',mom)
39 -  figure(1)
40 -  hist(mom)
41 -  xlabel('Moment for the steel beam (kN.m)','fontsize',12)
42 -  ylabel('Frequency','fontsize',12)
43 -  title('Histogram for Moment of Steel Girder.')
44 -  grid
45 -  Max_mom=max(mom)
46 -  Min_mom=min(mom)
47 -  M_mom=mean(mom)
48    % Variance of mom = v_mom
49 -  v_mom=var(mom,1)
50    % standard deviation value of mom = S_mom
51 -  S_mom=std(mom)
52    % Coefficient of Skewness of mom = B1_mom
53 -  B1_mom=skewness(mom)
54    % Coefficient of Kutosis of mom = B2_mom
55 -  B2_mom=kurtosis(mom)
```

Table A-3. Matlab code to generate a random sample, plot histogram, and determine the statistical properties for the mid-span deflection due to the second scenario.

```
1 -    clc
2      % Plot the Histogram for the Deflection of Steel Girder and Determine the
3      % Four Moments for the Data Obtianed Due to Variance in Applied Load and their position
4      % Uniform Self Weight = WD
5      % Concentrated Live Load = FL
6      % Concentrated superimposed Load = FD
7      % Modulus of Elastisity = E
8      % Moment of Inertia = I
9      % Length of the Beam = L
10     % Distance from the support to the first point load at left = a1
11     % Distance from the support to the first point load at right = b1
12     % Distance from the support to the second point load at left = a2
13     % Distance from the support to the second point load at right = b2
14 -   N=10000  % Sample size in Monte Carlo experiment for deflection results.
15 -   y=zeros(1,N) % Initialize a vector to store deflection of the beam "y"
16 -   for i=1:N
17         % The mean of moment of inertia = I
18 -       I=7.867*10^(-4)
19         % The mean of modulus of Elastisity = E
20 -       E=1986*10^(5)
21         % The mean length of beam = L
22 -       L = 9
23         % Log normal model to simulate random variance in the a1,b1,a2,b2
24     % for the first point load
25 -     m_a1 = 3 % mean of a1
26 -   v_a1 = 0.0441 % variance of a1
27 -   mu_a1 = log((m_a1^2)/sqrt(v_a1+m_a1^2))% mean for the log normal distribution
28 -   sigma_a1 = sqrt(log(v_a1/(m_a1^2)+1))%standard deiviation for the log normal distribution
29 -   [M,V]= lognstat(mu_a1,sigma_a1)
30 -   a1=lognrnd(mu_a1,sigma_a1)
31     %%%%%%%%%
32 -    m_b1 = 6 % mean of b1
33 -   v_b1 = 0.1764 % variance of b1
34 -   mu_b1 = log((m_b1^2)/sqrt(v_b1+m_b1^2))% mean for the log normal distribution
35 -   sigma_b1 = sqrt(log(v_b1/(m_b1^2)+1)) % standard deiviation for the log normal distribution
36 -   [M,V]= lognstat(mu_b1,sigma_b1)
37 -   b1= lognrnd(mu_b1,sigma_b1)
38     % for the second point load
39 -    m_a2 = 6 % mean of a2
40 -   v_a2 = 0.1764 % variance of a2
41 -   mu_a2 = log((m_a2^2)/sqrt(v_a2+m_a2^2))% mean for the log normal distribution
42 -   sigma_a2 = sqrt(log(v_a2/(m_a2^2)+1))%standard deiviation for the log normal distribution
43 -   [M,V]= lognstat(mu_a2,sigma_a2)
44 -   a2= lognrnd(mu_b1,sigma_b1)
45     %%%%%%%%%
46 -    m_b2 = 3 % mean of b2
47 -   v_b2 = 0.0441 % variance of b2
48 -   mu_b2 = log((m_b2^2)/sqrt(v_b2+m_b2^2))%mean for the log normal distribution
49 -   sigma_b2 = sqrt(log(v_b2/(m_b2^2)+1))%standard deiviation for the log normal distribution
50 -   [M,V]= lognstat(mu_b2,sigma_b2)
51 -   b2=lognrnd(mu_b2,sigma_b2)
52     % Normal model to simulate random variance in uniform dead load WD
53 -   WD=normrnd(2.912,0.233)
54     % Normal model to simulate random variance in concentrated dead load FD
55 -   FD=normrnd(122.323,9.786)
56     % Gumbel model to simulate random variance in uniform live load WL
57 -   mu_FL=56.727 % mu_FL = location parameter for Gumbel distribution
58 -   sigma_FL=4.634 % sigma_FL= scale parameter
59 -   FL=-evrnd(-mu_FL,sigma_FL)
60     % deflection due to uniform load = y1
61 -   y1=((5*(WD)*(L^4))/(384*E*I))
62     % deflection due to first point load = y2
63 -   y2=(((FD+FL)*(a1^2)*(b1^2))/(3*E*I*L))
64     % deflection due to second point load = y3
65 -   y3=(((FD+FL)*(a2^2)*(b2^2))/(3*E*I*L))
66     % the total deflection = y
67 -   y(1,i)=((y1+y2+y3)*1000)
68 -   end
69 -   xlswrite('load vary_def.xlsx',y)
70 -   figure(1)
71 -   hist(y)
72 -   xlabel('Deflection for the steel beam (mm)','fontsize',12)
73 -   ylabel('Frequency','fontsize',12)
74 -   title('Histogram for Deflection of Steel Girder.')
75 -   grid
76 -   Max_y=max(y)
77 -   Min_y=min(y)
78 -   M=mean(y)
79     % Variance of y = v
80 -   v=var(y,1)
81     % standard deviation value of y = S
82 -   S=std(y)
83     % Coefficient of Skewness of y = B1
84 -   B1=skewness(y)
85     % Coefficient of Kutosis of y = B2
86 -   B2=kurtosis(y)
```

Table A-4. Matlab code to generate a random sample, plot histogram, and determine the statistical properties for the applied moment of the girder due to the second scenario.

```matlab
1 -   clc
2     % Plot the Histogram for the Moment of Steel Girder and Determine the
3     % Four Moments for the Data Obtianed Due to Variance in Applied Load and their position
4     % Uniform Self Weight = WD
5     % Concentrated Live Load = FL
6     % Concentrated superimposed Load = FD
7     % Modulus of Elastisity = E
8     % Moment of Inertia = I
9     % Length of the Beam = L
10    % Distance from the support to the first point load at left = a1
11    % Distance from the support to the first point load at right = b1
12    % Distance from the support to the second point load at left = a2
13    % Distance from the support to the second point load at right = b2
14 -  n=10000  % Sample size in Monte Carlo experiment for moment results.
15 -  mom=zeros(1,n) % Initialize a vector to store moment of the beam "mom"
16 -  for i=1:n
17      % The mean length of beam = L
18 -      L = 9
19      % Distance from supports to applied loads in m
20      % for the first point load
21 -      m_a1 = 3 % mean of a1
22 -      v_a1 = 0.0441 % variance of a1
23 -      mu_a1 = log((m_a1^2)/sqrt(v_a1+m_a1^2))% mean for the log normal distribution
24 -      sigma_a1 = sqrt(log(v_a1/(m_a1^2)+1))%standard deiviation for the log normal distribution
25 -      [M,V]= lognstat(mu_a1,sigma_a1)
26 -      a1=lognrnd(mu_a1,sigma_a1)
27      % Normal model to simulate random variance in uniform dead load WD
28 -      WD=normrnd(2.912,0.233)
29      % Normal model to simulate random variance in concentrated dead load FD
30 -      FD=normrnd(122.323,9.786)
31      % Gumbel model to simulate random variance in uniform live load WL
32 -      mu_FL=56.727 % mu_FL = location parameter for Gumbel distribution
33 -      sigma_FL=4.634 % sigma_FL= scale parameter
34 -      FL=-evrnd(-mu_FL,sigma_FL)
35      % Moment in the middle of the girder due to uniform and concenterated load
36 -      mom(1,i)=(((WD*L^2)/8)+((FL+FD)*a1))
37 -  end
38 -  xlswrite('load vary.xlsx',mom)
39 -  figure(1)
40 -  hist(mom)
41 -  xlabel('Moment for the steel beam (kN.m)','fontsize',12)
42 -  ylabel('Frequency','fontsize',12)
43 -  title('Histogram for Moment of Steel Girder.')
44 -  grid
45 -  Max_mom=max(mom)
46 -  Min_mom=min(mom)
47 -  M_mom=mean(mom)
48    % Variance of mom = v_mom
49 -  v_mom=var(mom,1)
50    % standard deviation value of mom = S_mom
51 -  S_mom=std(mom)
52    % Coefficient of Skewness of mom = B1_mom
53 -  B1_mom=skewness(mom)
54    % Coefficient of Kutosis of mom = B2_mom
55 -  B2_mom=kurtosis(mom)
```

Table A-5. Matlab code to generate a random number, plot histogram, and determine the statistical properties for maximum allowable deflection.

```
1 -    clc
2      %%%%%% Plot the Histogram for the Max Allowable Deflection of Steel Girder and Determine the
3      % Four Statistical Moments for the Data Obtianed
4      % Max allowable deflection= y
5 -    n=10000  % Sample size in Monte Carlo experiment for deflection results
6 -    y=zeros(1,n)
7 -    for i=1:n
8          % Generate random numbers for beam length
9 -          m = 9 % mean of beam length random values in m
10 -     v = 0.3969 % variance of beam length random values
11 -     mu = log((m^2)/sqrt(v+m^2))% mean for the log normal distribution
12 -     sigma = sqrt(log(v/(m^2)+1)) % standard deiviation for the log normal distribution
13 -     [M,V]= lognstat(mu,sigma)
14 -     L= lognrnd(mu,sigma)
15 -     y(1,i)=L*1000/240
16 -    end
17 -    xlswrite('Max.all.def.xlsx',y)
18 -    figure(1)
19 -    hist(y)
20 -    xlabel('Max. allowable deflection for the steel beam (mm)','fontsize',12)
21 -    ylabel('Frequency','fontsize',12)
22 -    title('Histogram for Max. Allowable Deflection of Steel Girder.')
23 -    grid
24 -    Max_y=max(y)
25 -    Min_y=min(y)
26 -    M_y=mean(y)
27     % Variance of y = v_y
28 -    v_y=var(y,1)
29     % standard deviation value of y = S_y
30 -    S_y=std(y)
31     % Coefficient of Skewness of y = B1_y
32 -    B1_y=skewness(y)
33     % Coefficient of Kutosis of y = B2_y
34 -    B2_y=kurtosis(y)
```

Table A-6. Matlab code to generate a random number, plot histogram, and determine the statistical properties for M_n.

```
1 -    clc
2      %%%%%% Plot the Histogram for the Elastic Moment My as the Resistance of Steel Girder
3      % and Determine the
4      % Four Moments for the Data Obtianed
5      % Depth of beam =d
6      % Moment of inertia = I
7      % Elastic modulus section = S
8      % Yield strength for web MPa = FyW
9      % Yield strength for flange MPa = FyF
10     % Total yield strength for beam = Fy
11     % Elastic moment My = Fy * S
12 -   N=10000  % Sample size in Monte Carlo experiment for moment results
13 -   My=zeros(1,N)
14 -   for i=1:N
15         % Generate Random Variables for S in mm^3
16         % Normal model to simulate random variance for I in mm^4
17 -       I=normrnd(7867*10^(5),2753*10^(4))
18 -       d=normrnd(365,1.61)
19 -       c=d/2
20 -       S=I/c
21          % Generate Random Variables for yield stress in N/mm^2
```

```matlab
22 -          FyF=normrnd(297.3,16.8)
23 -          FyW=normrnd(307.3,16.8)
24 -          Fy=(FyF+FyW)/2
25 -          My(1,i)=Fy*S*10^(-6)
26 -      end
27 -      xlswrite('Elastic moment.xlsx',My)
28 -      figure(1)
29 -      hist(My)
30 -      xlabel('Elastic Moment (kN.m)','fontsize',12)
31 -      ylabel('frequency','fontsize',12)
32 -      title('Histogram of Data My.')
33 -      grid
34 -      Max_My=max(My)
35 -      Min_My=min(My)
36 -      M_My=mean(My)
37        % Variance of My = v_My
38 -      v_My=var(My,1)
39        % standard deviation value of My = S_My
40 -      S_My=std(My)
41        % Coefficient of Skewness of My = B1_My
42 -      B1_My=skewness(My)
43        % Coefficient of Kutosis of My = B2_My
44 -      B2_My=kurtosis(My)
```