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Effect of Thickness Variable on the Bending Analysis of Rotating Functionally polymer Graded Carbon Nanotube Reinforced Cylindrical Panels

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ABSTRACT

This study offers the elastic response of the variable thickness functionally graded (FG) by single walled carbon nanotubes reinforced composite (CNTRC) moderately thick cylindrical panels under rotating and transverse mechanical loadings. It's considered that, three kinds of distributions of carbon nanotubes which are uniaxial aligned in the longitudinal direction and two functionally graded in the transverse direction of the cylindrical panels. Depending on first order shear deformation theory (FSDT), the governing equations can be derived. The partial differential equations are solved by utilizing the technique of finite element method (FEM) with a program has been built by using FORTRAN 95. The results are calculated to investigate the influence of the variable thickness, geometric parameters, rotating velocity, carbon nanotubes (CNTs) volume fraction for different boundary conditions on the non-dimensional deflection of the cylindrical panels. A comparison study has been carried out between the results of present study and that available in the open literature and found very good correspondence between the two results.

Keywords: variable thickness, cylindrical panels, bending analysis, functionally graded, carbon nanotubes.

تأثير تغيير السمك على تحليل الانحناء للالواح الاسطوانية الدواره المدعمه بأنابيب النانوكاربون بوليمر بشكل متدرج

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الخلاصة

هذه الدراسة عرضت السلوك المرن للالواح الاسطوانية متغيره السمك المتدرجه المعززة بانابيب النانوكاربون احادية الجدار تحت تأثير الاحمال الميكانيكية الدورانية والعرضية. تم الاخذ بنظر الاعتبار ثلاثة انواع من اشكال التوزيع لانابيب النانوكربون خلال اتجاه السمك للالواح الاسطوانية. تم اشتقاق معادلات الحركة بالاعتماد على نظرية تشوه القص من الدرجة الاولى . استخدمت طريقة

العناصر المحددة لحل المعادلات التفاضلية الجزئية من خلال بناء برنامج بلغة الفورتران (95). تم استعراض النتائج العددية لبيان تأثير تغير السمك للالواح الاسطوانية ، عوامل الشكل الهندسي ، السرعة الدورانية باختلاف الشروط الحديه،النسبة الحجمية للانابيب النانوكاربون على سلوك الانحناء للالواح الاسطوانية .تم اجراء دراسة مقارنة مع نتائج الدراسات السابقة ووجد بان هناك توافق جيد جدا مع تلك النتائج .

1. INTRODUCTION

Cylindrical panel structures which has variable thickness are remarkable kind of modernistic mechanical structures, considerably utilized in most industries such aerospace industry, rotating parts of turbo machinery, space shuttle and in the industry of the remarkable engineering structures. The search of elastic response of the rotating parts with variable thickness is very important to guaranty an effective and credible design. Furthermore, the utilize of the variable thickness mechanical parts assists to decrease the weight of the engineering structures and get better the economic use for the materials. In the last decade, due to its important for this kind of structures, there have been growing studies in the field of vibration, buckling and static behaviour. The vibration, static and buckling behavior for the variable thickness cylindrical panels have been presented by many researchers. For example, the effect of variable thickness for the twisted curved cylindrical shell panels on the vibration behaviour by utilizing the virtual work principle and Ritz technique presented by **Sakiyama, et al., 2002.** Also, **Tornabene, et al., 2016**, utilized the local generalized differential quadrature technique to investigate the effect of variable thickness on the natural frequencies for the doubly-curved shells. **Özakça, et al., 2003,** used the integrating finite strips to determine the influence of variable thickness on the plates on the buckling load of the prismatic folded plates. **Sofiyev and Aksogan, 2004**, utilized the Galerkin method and the Rayleigh-Ritz procedure to investigate the buckling of variable thickness conical shell under the influence of transient loads. The vibration behaviour for the twisted conical shells together with variable thickness investigated by **Hu, et al., 2002**. The governing equations for the shell derived based on the Rayleigh-Ritz method. **Duan and Koh, 2008** utilized the analytical solution to obtain the transverse vibration for the circular cylindrical shells with the variable thickness. **Chen, et al., 2011,** investigated the buckling for the cylindrical shells with the variable thickness subjected to external pressure.

The utilizing for the functionally graded materials (FGMs) in the last few years have earns heavy interest in most engineering applications. In the following, it has been focused on the studies in static and dynamic analyses of FG variable thickness shells. **Shariyat and Asgari, 2013,** studied the nonlinear buckling and postbuckling behaviour of FG cylindrical shells with variable thickness under the effect of thermal loads. **Jabbari, et al., 2015**, used the higher order shear deformation theory to investigate thermo-elastic behavior of FG rotating variable thickness cylindric with variable thickness under thermal and disturb pressure. Also, the elastic analyses of FG variable thickness cylindrical shells subjected to internal pressure performed by **Ghannad, et al., 2013**. The elastic response for FG rotating variable thickness cylinder under asymmetric arbitrary pressure loading studied by ,**Nejad, et al., 2015**. The semi-analytical solution utilized by, **Dai and Dai, 2016** to study thermo-elastic behaviour for FG variable thickness rotating circular disk with angular speed. **Bacciocchi, et al., 2016,** used generalized differential quadrature technique to investigate the free vibration behavior of composite plates and shells with variable thickness. **El-Kaabazi and Kennedy, 2012,** used the

wittrick-williams procedure to obtain the vibration modes and natural frequencies for cylindrical shells with variable thickness.

In last few years, Carbon nanotubes as the reinforcing materials in the composite structures have encouraged a lot of researchers. This is thanks to superior mechanical properties as compared with carbon fibers, **Wang, et al., 2014**. In the following, it has been focused on the static and dynamic behaviour of CNTs reinforced FG cylindrical shell panels. **Zhang, et al., 2014b,** utilized the FSDT to analyze transverse displacement and natural frequencies for the cylindrical panels reinforced by CNTs. The technique of KP-Ritz method used by, **Zhang, et al., 2014a** to study the nonlinear behaviour of carbon nanotube reinforced FG cylindrical shell panels. **Lei, et al., 2015,** used the element-free method to estimate the vibration behaviour of CNTs reinforced FG rotating cylindrical shell panels with constant thickness. Also**, Lei, et al., 2016**, performed an analysis for the rotating laminated carbon nanotubes reinforced FG cylindrical panels by using element-free procedure.

The FSDT utilized by, **Mirzaei and Kiani, 2016** to present the vibration behaviour for the cylindrical panels reinforced by FG with single wall carbon nanotubes. **Wang, et al., 2017,** studied the vibration behaviour of functionally graded CNTs-reinforced doubly-curved shell panels by utilizing the semi-analytical procedure. **Alibeigloo, 2016**, utilized the state space method to show the thermoelastic behaviour of FG (CNTs) reinforced cylindrical panels subjected to thermal and mechanical loads. **Mehar and Panda, 2017,** presented the nonlinear deflection for functionally graded CNTs-reinforced composite shell panels under the influence of thermal and mechanical loads.

It can be observed that the elastic analyses of the variable thickness cylindrical panel reinforced by CNTs has not been so far investigated. Hence, this work will concentration on the studying the investigation gap. Moreover, the cylindrical panel is undergone to combine the rotating and symmetric external load. Based on the FSDT, the governing equations are arranged in linear form. By applying the principal of minimum total potential energy, the resulting partial differential equations are solved by utilizing the FEM. In numerical results, the influence of the variable thickness, geometric parameters, rotational speed as well as CNTs volume fraction for two types of boundary conditions on the non-dimensional transverse displacement are researched.

2. GEOMETRIC MODEL FOR THE CNTS CYLINDRICAL PANEL

In present study, a rotating cylindrical panel having longitudinal length L ,lateral length b , radius R having thickness h and rotating at constant angular velocity Ω about the x as shown in **Fig.1**.
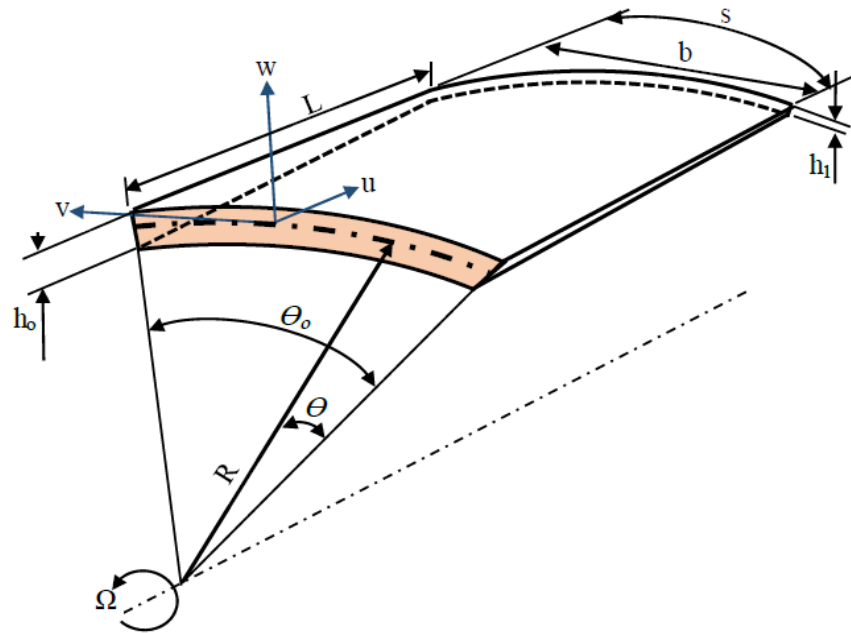
Figure 1. Geometric model for the CNTRC cylindrical panel.

Consider that the cylindrical panel thickness changes in the x direction as follows, **Thang, et al., 2016**:

$$h(x)=h_O\left[1.0+c\left(\frac{x}{b}\right)^d\right]$$ (1)

Where h_O represents the thickness at $x=0$ of the cylindrical panel and d denotes nonlinear parameter for the varying thickness of the CNT cylindrical panel. For instance, if the value of, $d=0$, then the panel thickness is constant. Also, if the value of parameter, $d=1$, then the thickness for the panel varies in a linear from h_O at $x=0$ to h_1 at $x=L$. Also, if $d=0.5$ or $d=2$ the panel thickness varies parabolic in the x direction.

The expression c can be specified as follows

$c=\dfrac{h_1-h_o}{h_o}$, Where h_1 represents the panel thickness at $x=L$.

3. CARBON NANOTUBE REINFORCED CYLINDRICAL PANELS

In this work, as shown in **Fig.2** three kinds of carbon nanotubes reinforced cylindrical panels. The symbol UD denotes the uniform distributions, FG_V and FG_X the functionally graded distributions for the carbon nanotubes in the transverse direction of the cylindrical panels.
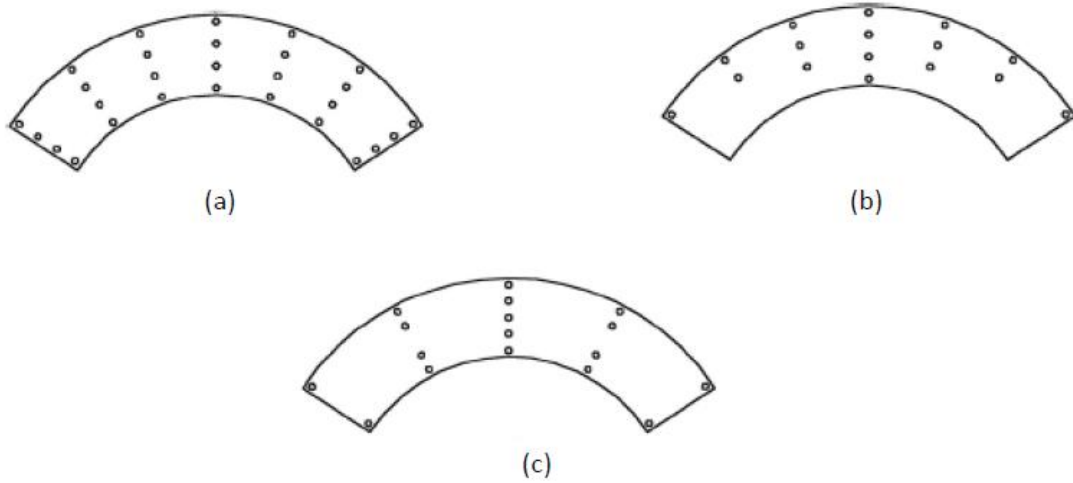
Figure 2. Configurations of CNTs for the FG-CNTRC cylindrical panels. (a)UD CNTRC cylindrical panel ;(b) FG-V CNTRC cylindrical panel; (c) FG-X CNTRC cylindrical panel.

The rule of mixture utilized to estimate the effective material properties by inserting the CNT efficiency parameters (η) and it can be written as, **Zhu, et al., 2012**.

$$E_{11}=\eta_1 V_{CNT} E_{11}^{CNT} + V_m E^m$$

$$\frac{\eta_2}{E_{22}}=\frac{V_{CNT}}{E_{22}^{CNT}}+\frac{V_m}{E^m} \qquad\qquad (2)$$

$$\frac{\eta_3}{G_{12}}=\frac{V_{CNT}}{G_{12}^{CNT}}+\frac{V_m}{G^m}$$

Where G_{12}^{CNT} and E_{11}^{CNT} , E_{22}^{CNT} stands for the shear modulus and young's moduli for CNTs respectively. Also, G^m and E^m stands for the shear modulus and young's modulus for the isotropic matrix.

The uniform and two kinds of FG distributions of the carbon nanotubes through the transverse direction of the cylindrical panels described in **Fig.2** are considered as following

$$V_{CNT}=V_{CNT}^* \qquad\qquad (UD)$$

$$V_{CNT}(z)=\left(1+\frac{2z}{h}\right)V_{CNT}^* \qquad (FG\_V) \qquad (3)$$

$$V_{CNT}(z)=2\left(\frac{2|z|}{h}\right)V_{CNT}^* \qquad (FG\_X)$$

As well, by utilizing the rule of mixture, Poisson's ratio and the mass density can be evaluated as

$$v_{12} = V_{CNT}^* v_{12}^{CNT} + V_m v^m$$

$$\rho = V_{CNT}^* \rho^{CNT} + V_m \rho^m$$

Where v_{12}^{CNT} and v^m stands for Poisson's ratio of CNTs and matrix respectively.

4. PROBLEM FORMULATION

4.1 Governing equations

In the present research, it is considered moderately thick CNTRC cylindrical panels. Depending on the FSDT, the displacement variables $(u, \theta, w)^T$ through the panel domain is expressed as, **Reddy JN, 2006.**

$$u(x, \theta, z) = u_0(x, \theta) + z\varphi_x(x, \theta)$$
$$v(x, \theta, z) = v_0(x, \theta) + z\varphi_\theta(x, \theta)$$
$$w(x, \theta, z) = w_0(x, \theta)$$

(4)

Where u_o , v_o , w_o, φ_x and φ_θ stands for the displacement vectors at the mid-plane of the panels.

The linear strain-displacement relations are considered as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{1}{R}\dfrac{\partial v_o}{\partial \theta} + \dfrac{w_o}{R} \\ \dfrac{1}{R}\dfrac{\partial u_o}{\partial \theta} + \dfrac{\partial v_o}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \dfrac{\partial \varphi_x}{\partial x} \\ \dfrac{1}{R}\dfrac{\partial \varphi_\theta}{\partial \theta} \\ \dfrac{1}{R}\dfrac{\partial \varphi_x}{\partial \theta} + \dfrac{\partial \varphi_\theta}{\partial x} \end{Bmatrix}$$

(5)

$$\begin{Bmatrix} \gamma_{\theta z} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \varphi_\theta + \dfrac{1}{R}\dfrac{\partial w_o}{\partial \theta} - \dfrac{1}{R} v_o \\ \varphi_x + \dfrac{\partial w_o}{\partial x} \end{Bmatrix}$$

Then, the constitutive equations are expressed in the following

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{\theta\theta} \\ \sigma_{x\theta} \\ \sigma_{\theta z} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \\ \gamma_{\theta z} \\ \gamma_{xz} \end{Bmatrix}$$

(6)

Where

$$Q_{11} = \frac{E_{11}}{1 - v_{12}v_{21}}, \qquad Q_{22} = \frac{E_{22}}{1 - v_{12}v_{21}}, \qquad Q_{12} = \frac{v_{21}E_{11}}{1 - v_{12}v_{21}}$$

$$Q_{66} = G_{12}, \qquad\qquad Q_{44} = G_{23}, \qquad\qquad Q_{55} = G_{13}$$

As aforementioned previously, in this research the FEM is utilized to describe the governing equations in the locative domain and the scheme for it demonstrates in **Fig.3**. Thus, the motion equations for the eth element can be derives by applying the minimum total energy as follows, **Reddy JN, 2006.**
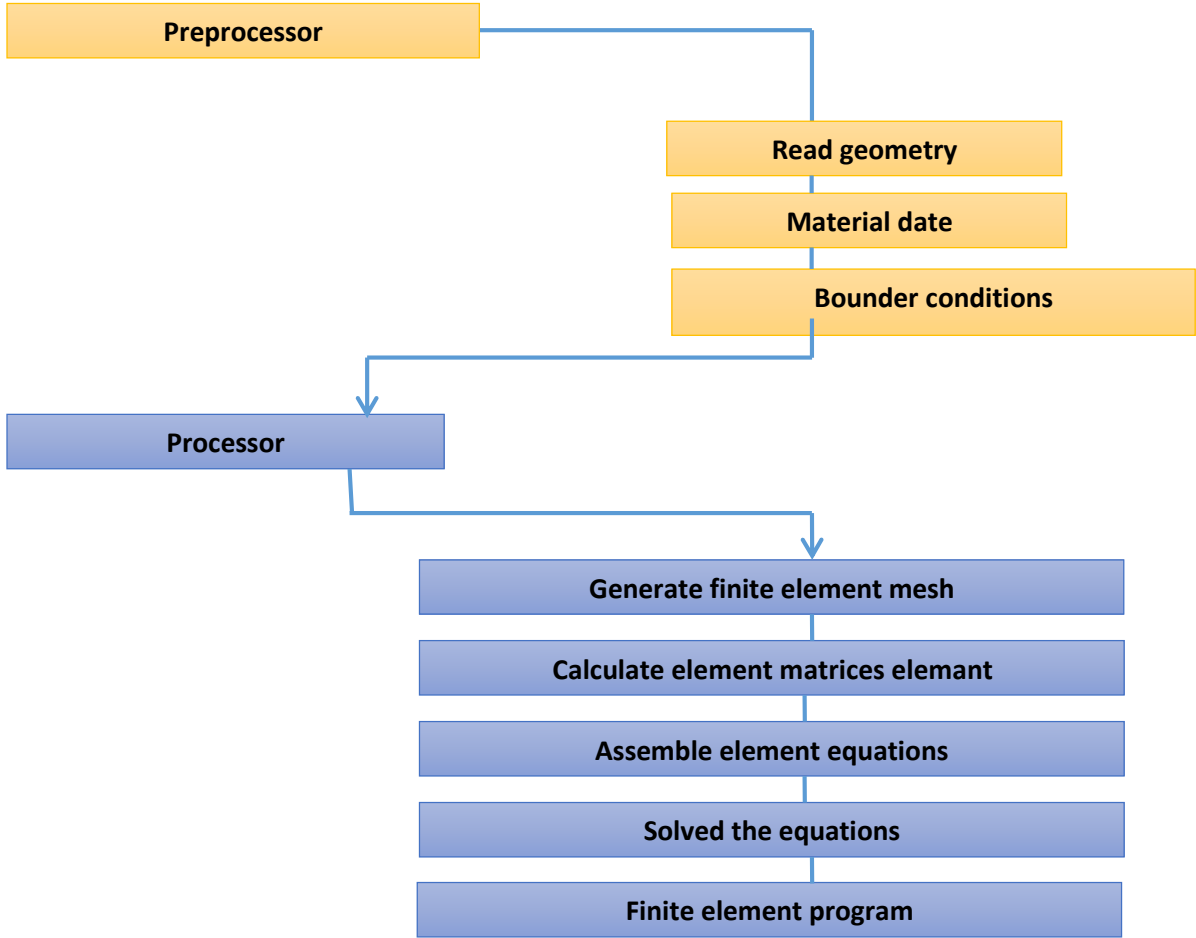

Figure 3. Finite element program.

$$\delta\prod^{(e)}=\delta U^{(e)}-\delta V^{(e)}=0 \tag{7}$$

Here, $\delta\prod^{(e)}$, $\delta U^{(e)}$ and $\delta V^{(e)}$ stands for the potential energy, strain energy and the work done respectively.

$$\delta U^e = \int_{-h(x)/2}^{h(x)/2}\int_{A^e}\left(\sigma_{xx}\delta\varepsilon_{xx}+\sigma_{\theta\theta}\delta\varepsilon_{\theta\theta}+\sigma_{x\theta}\delta\gamma_{x\theta}+\sigma_{xz}\delta\gamma_{xz}+\sigma_{\theta z}\delta\gamma_{\theta z}\right)dz\,dA \tag{8}$$

$$\delta V^e = -\int_{-h(x)/2}^{h(x)/2}\int_{A^e}\rho(x)R^2\Omega^2\delta w\,dz\,dA - \int_{A^e}q_z\delta w\,dA \tag{9}$$

This study is utilized four-node Lagrange element with twenty degrees of freedom per element $\left(U^e,\ V^e,\ W^e,\ \varphi_x^e,\ \varphi_\theta^e\right)$ to describe the locative domain. The domain vectors $\left(u^e,\ v^e,\ w^e,\ \varphi_x^e,\ \varphi_\theta^e\right)$ can be estimated in each element by utilizing the interpolation function as follows, **Reddy JN, 2006.**

$$u^e(x,\theta) = \sum_{j=1}^{4} U_j^e \Psi_j^e(x,\theta)$$

$$v^e(x,\theta) = \sum_{j=1}^{4} V_j^e \Psi_j^e(x,\theta) \tag{10}$$

$$w^e(x,\theta) = \sum_{j=1}^{4} W_j^e \Psi_j^e(x,\theta)$$

$$\varphi_x^e(x,\theta) = \sum_{j=1}^{4} \phi_{xj}^e \Psi_j^e(x,\theta)$$

Here, $U^e,\ V^e,\ W^e,\ \varphi_x^e,\ \varphi_\theta^e$ stands for the degrees of freedom for the eth element and Ψ_j^e are the two-dimensional shape functions. By substituting Eq.(8-9) and (4-6) into Eq.(7) and utilizing integration by parts, the discretized governing equations of motion for the eth CNT shell element are obtained as follows,

$$k^e d^e = F^e \tag{11}$$

Here, k^e, d^e and F^e are the stiffness matrix, displacement matrix and load vector for the eth element.

$$k^e = \begin{bmatrix} k_{11}^e & k_{12}^e & k_{13}^e & k_{14}^e & k_{15}^e \\ k_{21}^e & k_{22}^e & k_{23}^e & k_{24}^e & k_{25}^e \\ k_{31}^e & k_{32}^e & k_{33}^e & k_{34}^e & k_{35}^e \\ k_{41}^e & k_{42}^e & k_{43}^e & k_{44}^e & k_{45}^e \\ k_{51}^e & k_{52}^e & k_{53}^e & k_{54}^e & k_{55}^e \end{bmatrix} \tag{12}$$

$$d^e = \begin{Bmatrix} U^e \\ V^e \\ W^e \\ \Psi^e_x \\ \Psi^e_\theta \end{Bmatrix} , \quad \mathbf{F}^e = \begin{Bmatrix} F^e_1 \\ F^e_2 \\ F^e_3 \\ F^e_4 \\ F^e_5 \end{Bmatrix} \tag{13}$$

In this research, the cylindrical panels reinforced by CNTs with simply supported (S) as well as clamped (C) edges are investigated. The considered essential boundary conditions are

Clamped (C): $u = 0$, $v = 0$, $w = 0$, $\varphi_x = 0$, $\varphi_\theta = 0$

Simply supported (S): $u = 0$, $v = 0$, $w = 0$

As soon as the assembling process has been completed and after applying the geometric boundary conditions, the equations of motion of the CNTRC moderately thick cylindrical panels in the matrix form as follows

$$Kd = F \tag{12}$$

Here, K , d and F are the global stiffness matrix, displacement vector and load vector respectively.

5. RESULTS AND DISCUSSION

In this part, bending analysis for CNTRC cylindrical panels is investigated by using finite element method. PMMA (poly methyl methacrylate), **Han and Elliott, 2007,** is selected as the matrix with mechanical properties for which are considered to be $v^m = 0.34$, $\rho^m = 1.15 \, g/cm^3$ and $E^m = 2.1 GPa$ at the temperature of room. Also in this work, single-walled carbon nanotubes are adopted as the reinforcements. In accordance with molecular dynamic (MD) simulation presented by, **Zhang and Shen, 2006**, the mechanical properties of the CWCNT are adopted from Reference, **Zhang and Shen, 2006** as, $E^{CNT}_{11} = 5.6466 TPa$, $E^{CNT}_{22} = 7.08 TPa$ and $G^{CNT}_{12} = 1.9445 TPa$. The parameters of efficiency for CNT η_i defined in eq. (1a-c) can be specified in accordance with the effective mechanical properties of carbon nanotube available, **Zhu, et al., 2012** through matching the modulus of elasticity E_{11} and E_{22} together with the counterpart evaluated in the rule of mixture. In the present research, the efficiency parameters are chosen as reference, **Han and Elliott, 2007** , for instance, $\eta_1 = 0.149$ and $\eta_2 = 0.934$ in the condition of $V^*_{CNT} = 0.11$ and $\eta_1 = 0.15$, $\eta_2 = 0.941$ in the condition of $V^*_{CNT} = 0.14$ and $\eta_1 = 0.149$, $\eta_2 = 1.381$ for the case of $V^*_{CNT} = 0.17$. Also, it's assumed that $\eta_3 = \eta_2$ and $G_{12} = G_{13} = G_{23}$. These amounts will be considered in the subsequent numerical examples. To prove the validation for the present work, an analysis for CNTRC cylindrical panels by **Zhang, et al., 2014** used mesh-free KP-Ritz technique is completed. Geometric dimensions and the

material properties of the cylindrical panel are taken as in the same reference above. The dimensionless central deflection $\left(\dfrac{w}{h}\right)$ is presented in **Table 1**.

Table1. Comparison of the dimensionless central deflection $\left(w/h\right)$ for the CNTRC cylindrical panels under the effect of uniform distribution pressure ($P_o = 0.1 MPa$)
(Zhang, et al., 2014).

|		V_{CNT}								
		0.11			0.14			0.17		
		Present work	Zhang et al., 2014	Error	Present work	Zhang et al., 2014	Error	Present work	Zhang et al., 2014	Error
SSSS	**UD**	1.113	1.115	0.18%	0.892	0.888	0.44%	0.728	0.727	0.1%
	FG_V	1.544	1.551	0.19%	1.254	1.253	0.079%	1.016	1.017	0.098%
	FG_X	0.773	0.770	0.38%	0.617	0.611	0.97%	0.504	0.500	0.79%
CCCC	**UD**	0.247	0.250	1.21%	0.203	0.202	0.49%	0.161	0.162	0.61%
	FG_V	0.340	0.348	2.3%	0.277	0.281	1.42%	0.223	0.228	2.19%
	FG_X	0.181	0.180	0.55%	0.150	0.147	2.0%	0.117	0.116	0.85%

It can be observed that there agreement between the results of this study with that presented in the literature. In accordance to accuracy and efficiency, discretization with 40×40 elements is utilized for all additional analysis.

6. RESULTS OF PRESENT WORK

Next, to show the proposed approach, CNTRC cylindrical panels under the effect of rotational velocity and uniform distribution pressure are investigated. **Figs 4** and **5** present the influence of the variable thickness on the non-dimensional central deflection for the simply supported and clamped boundary conditions respectively.
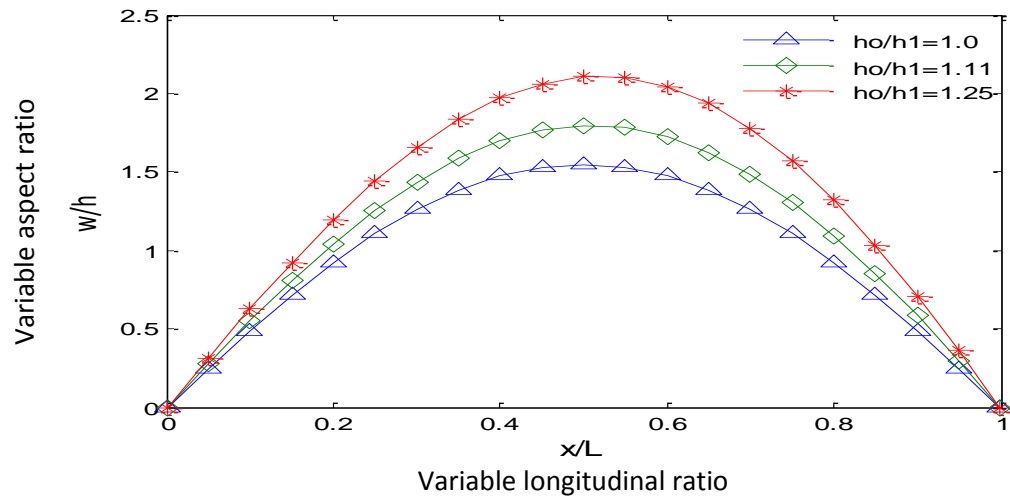
Figure 4. Effect of the variable thickness on the central deflection (w/h) of FG-CNTRC cylindrical panel along the centre line $(x, \theta/2)$ with four edges simply supported.
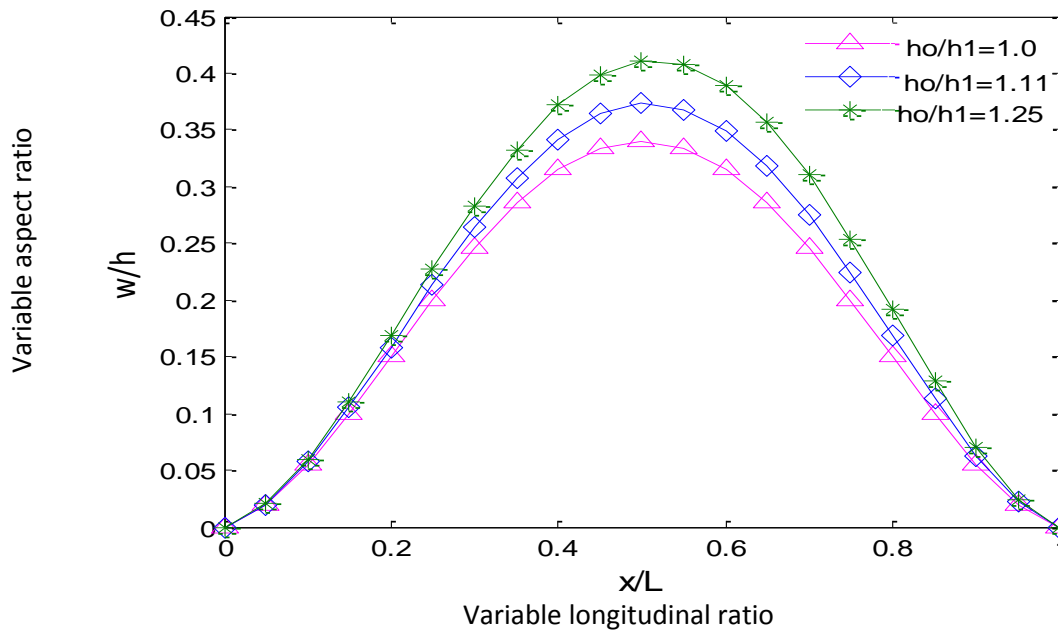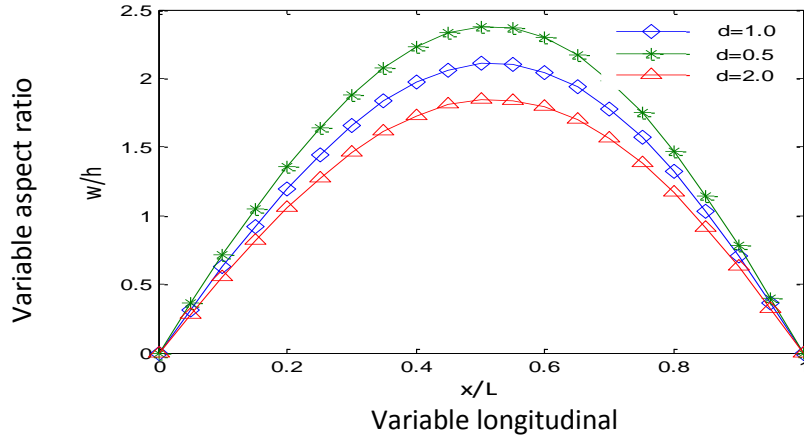

Figure 5. Influence of the variable thickness on the central deflection   of FG-CNTRC cylindrical panel along the center line   with four edges clamped.

It can be observed that the slightly decrease in the thickness lead to increase in the central deflection. Also, because of the constraint for the simply supported boundary conditions is weaker than that for the clamped boundary conditions, the central deflection of the CNTRC cylindrical panels in the case of clamped edges is smaller than that with all edges simply

supported. **Figs. 6** and **7** present the effect of parameter d on the non-dimensional central deflection for both two boundary conditions clamped and simply supported.

Figure 6. Influence of the parameter d on the central deflection of FG-CNTRC cylindrical panel along the centre line   with four edges simply supported.
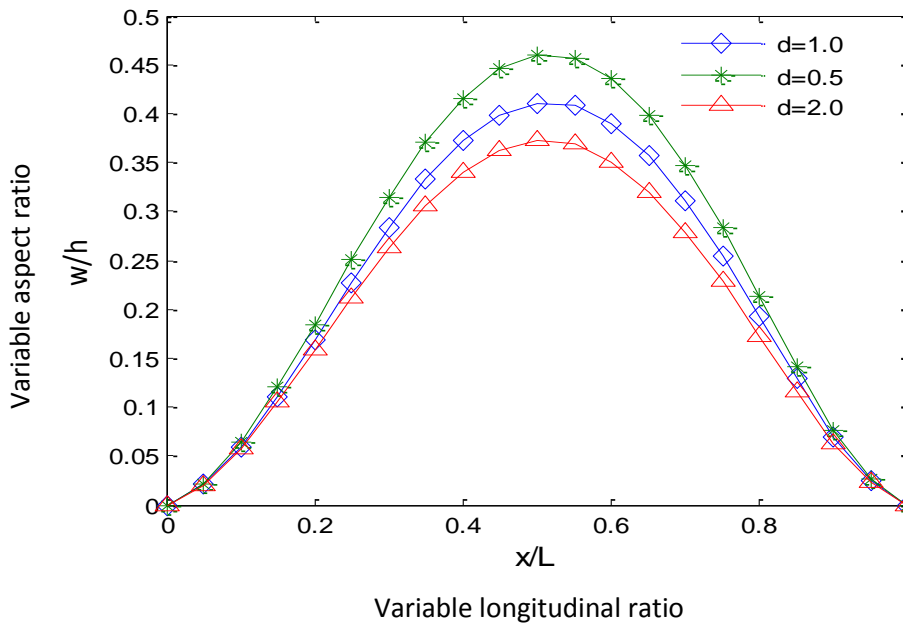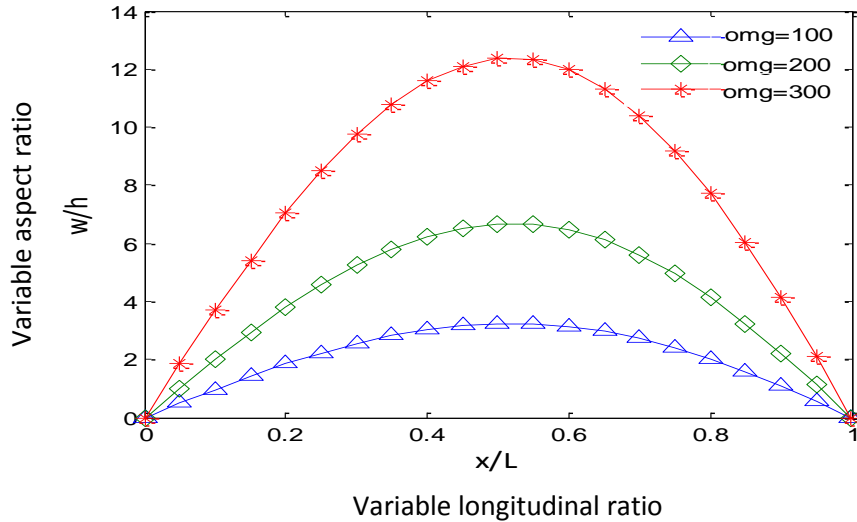

Figure 7. Influence of the parameter d on the central deflection   of FG-CNTRC cylindrical panel along the centre line   with four edges clamped.

It is observed that when d=0.5, the shape of the cylindrical panel is convex and the non-dimensional deflection goes to the maximum value. Also, as the value of parameter d=2.0, the shape of the cylindrical panel is concave and the central deflection takes the minimum

value, while when the parameter d=1.0, the thickness of the panel varies linearly and so the non-dimensional central deflection takes the mid value.
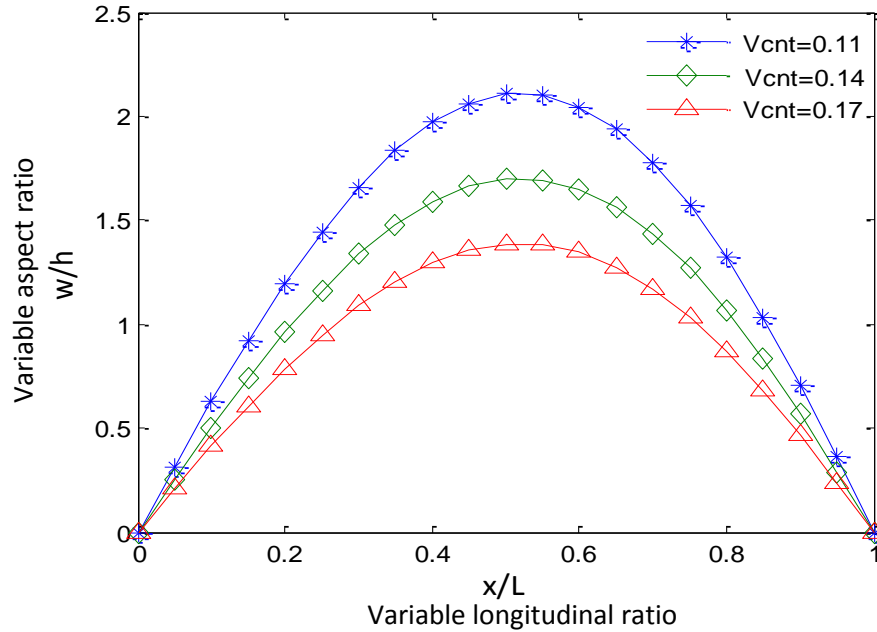

Figure 8. Influence of the rotational velocity on the central deflection of FG-CNTRC cylindrical panel along the centre line with four edges simply supported.
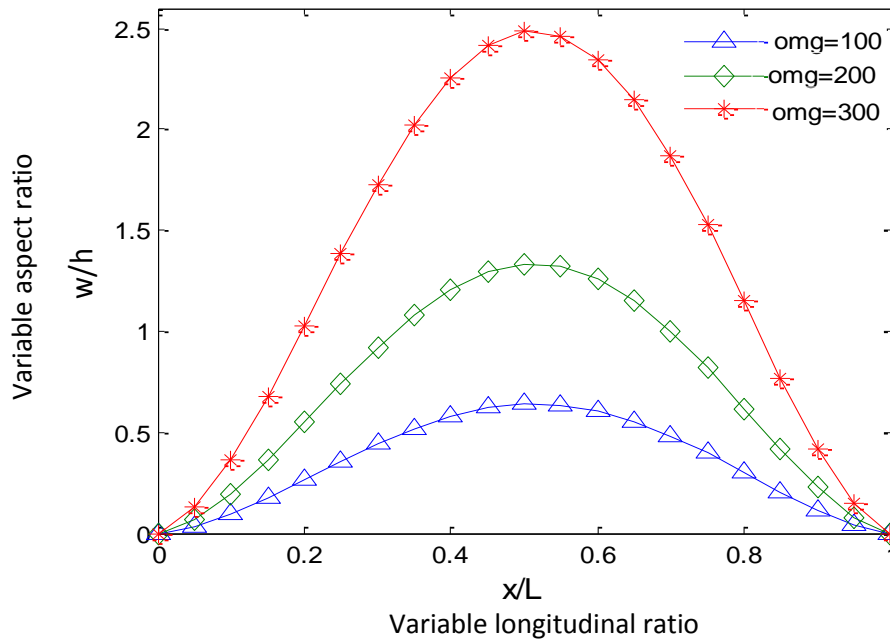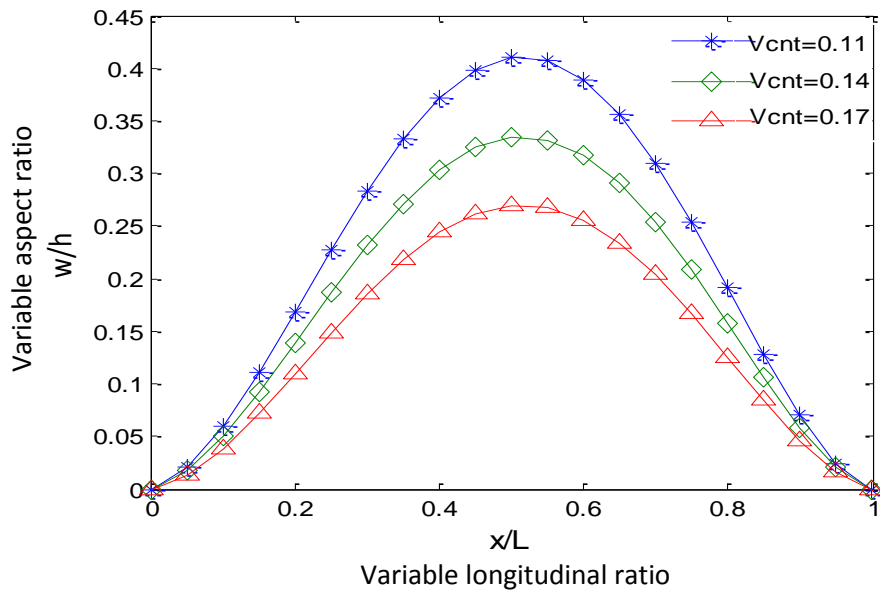

Figure 9. Influence of the rotational velocity on the central deflection of FG-CNTRC cylindrical panel along the centre line with four edges clamped.

From the observation of **Figs. 9** and **10**, it's clear that there is a rapid increase in the non-dimensional deflection when the rotational velocity increases from ($\Omega = 100 \ rad/\sec$) to ($\Omega = 300 \ rad/\sec$).

Figure 10. Influence of the volume fraction CNTs on the middle deflection of FG-CNTRC cylindrical panel along the centre line with four edges simply supported.

Figure 11. Influence of the volume fraction CNTs on the middle deflection of FG-CNTRC cylindrical panel along the center line with four edges clamped.

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 Figs 12 and **13** exhibit the influence for the volume fraction for the CNTs on the non-dimensional deflection.
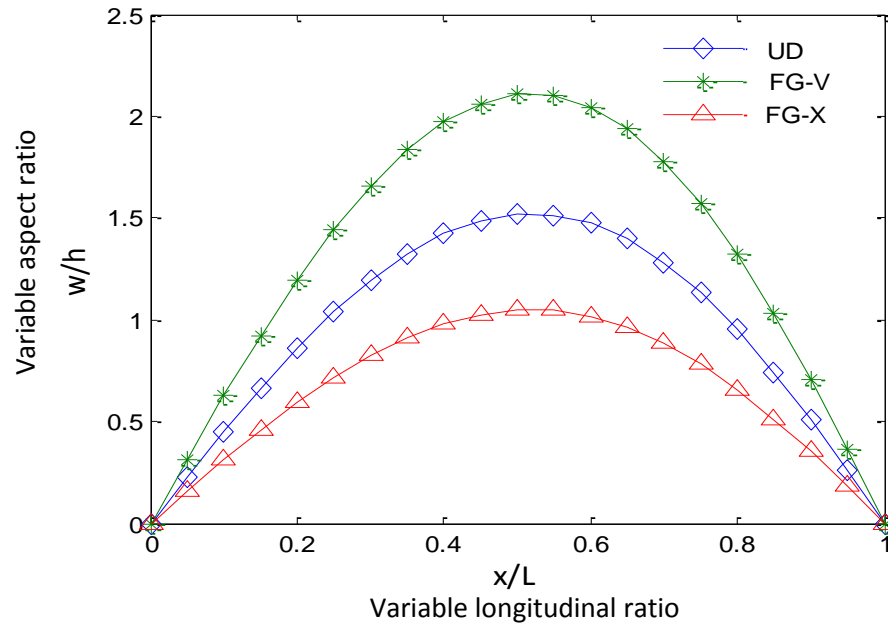

Figure 12. Influence of the CNTs distribution on the central deflection   of FG-CNTRC cylindrical panel along the centre line   with four edges simply supported
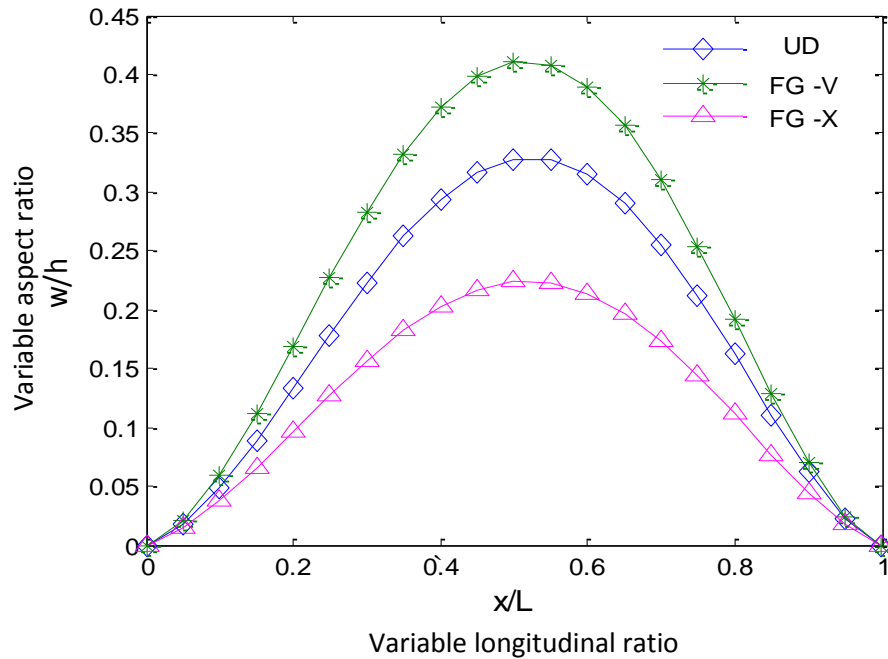

Figure 13. Effect of the CNTs distribution on the central deflection   of FG-CNTRC cylindrical panel along the centre line   with four edges clamped.

It's obtained that the non-dimensional central deflection of the cylindrical panels has lowest magnitudes when the volume fraction for the CNTs is higher, this is due to the increase in the stiffness of the CNTRC cylindrical panels with the increasing in the volume fraction of CNTs. Finally, it can be observed that from Figs. 12 and 13, the non-dimensional deflection for FG_V cylindrical panels have the highest magnitude, as compared with FG_X cylindrical panels is the lowest value.

7. CONCLUSIONS

The influence of the variable thickness on the flexural strength of the (CNTRC) cylindrical panels subjected to rotational velocity and symmetric distributed load is studied in the present work. The governing equations are formulated depend on the first-order shear deformation theory. The CNTs are graded in the transverse direction and the material properties are computed by employing modified rule of mixture. The FEM is employed to discretize the governing equations and a comparison studies are accomplished to guaranty the effectiveness of the present FEM. Numerical examples are employed to reveal the flexural strength characteristics for the CNTRC cylindrical panels. It can be concluded from the numerical examples:

- The variable thickness has a great influence on the bending behaviour for CNTRC cylindrical panels.
- Also, the type of constraint, simply supported or clamped four edges plays significant role on the central deflection of the cylindrical panels.
- The effect of the thickness non-uniform parameter "d" in the case of simply supported on the bending behaviour is greater than in the case of clamped four edges.
- The increasing in the rotational velocity leads to large increase in the amplitude of the central deflection.
- The volume fraction of the CNTs has considerable effect on the bending behaviour of the cylindrical panels.
- Also, the distribution type for the CNTs has significant influence on the flexural strength of the CNTs cylindrical panels.

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