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Robust Adaptive Sliding Mode Controller for a Nonholonomic Mobile Platform

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ABSTRACT

In this paper, a robust adaptive sliding mode controller is designed for a mobile platform trajectory tracking. The mobile platform is an example of a nonholonomic mechanical system. The presence of holonomic constraints reduces the number of degree of freedom that represents the system model, while the nonholonomic constraints reduce the differentiable degree of freedom. The mathematical model was derived here for the mobile platform, considering the existence of one holonomic and two nonholonomic constraints imposed on system dynamics. The partial feedback linearization method was used to get the input-output relation, where the output is the error functions between the position of a certain point on the platform and the desired path. The dynamic error model was considered uncertain and subjected to friction torques on the wheels. The adaptive sliding mode control was utilized to design a robust controller, that will force the platform to follow the desired trajectory. The simulation of the proposed controller was done via MATLAB to reveal the ability of the robust adaptive sliding mode controller applied as a trajectory tracker for various path shapes.
Keywords: Mobile platform, Non-holonomic constraints, Robust adaptive sliding mode controller, Partial feedback linearization.

تصميم مسيطر منزلق المتكيف المتين لمنصة التحرك الذاتي ذات القيود الغير التامة

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الخلاصة

في هذه الورقة البحثية، تم تصميم وحدة تحكم باستخدام المسيطر المنزلق لتتبع مسار منصة ذاتية التحرك. منصة ذاتية التحرك هي مثال على نظام ميكانيكي غيرتامة التقييد. إن وجود قيود تامة يقلل من درجة الحرية التي تمثل نموذج النظام ، في حين أن القيود غير التامة تقلل من درجة الحرية المختلفة. تم اشتقاق النموذج الرياضي هنا لمنصة ذاتية التحرك ، مع الأخذ في الاعتبار قيد تام واحدة وقيدين غير تامين مفروضين على ديناميكية النظام. تم استخدام طريقة التغذية الخلفية الجزئية للحصول على علاقة المدخلات والمخرجات ، حيث يكون الناتج هو وظائف الخطأ بين موضع نقطة معينة على المنصة والمسار المطلوب. واعتبر

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النموذج الديناميكي للخطأ غير مؤكد ، وتعرض لعزم إحتكاك الدوران على العجلات. تم استخدام المسيطر المنزلق المكييف لتصميم وحدة تحكم متينة ، والتي ستجبر المنصة على اتباع المسار المطلوب. تم إجراء محاكاة وحدة التحكم المقترحة عبر MATLAB للكشف عن قدرة المسيطر المنزلق المتين المكييف كمتتبع مسار لأشكال المسيرات المختلفة.
الكلمات الرئيسية: منصة التحرك الذاتية، المسيطر المنزلق المتكيف المتين، القيود التامة، القيود الغير تامة، , التغذية الخلفية الجزئية

1. INTRODUCTION

All Physical systems can be categorized into two categories: Linear systems and nonlinear systems. The nonlinear systems are classified into holonomic and nonholonomic systems according to types of their constraint**,** as showed by **Chung, 2004**.

In mechanics, the free rigid body is considered as the main primary system, and this topic was treated and studied to a great extent. **Baillieul, et al., 2003,** showed that the mobile platform is one of the most important applications on the nonholonomic mechanic systems and its tracking control is a challenging task.

The relation between control theory and the geometric mechanics is done with classical mechanical Lagrange formulation which describes the dynamic of the system in the presence of its constraints (holonomic and nonholonomic). The Lagrangian equation equal to the difference between kinetic energy and potential energy as demonstrated later in this paper. **Bloch, 2003,** showed that there is a clear, strong connection between mechanics and nonlinear control theory concerning with theory on moving mechanical system under constraints**.**

There are many types of nonlinear control applied to control the trajectory tracking of nonholonomic mobile platform. For example, backstepping based trajectory tracking control was applied to a four-wheel mobile robot by **Kumar and Sukavanam, 2008**; nonlinear controller was designed for mobile manipulator trajectory by **Said, et al., 2014,** and sliding mode real time, which was designed to mobile platform control system in the presence of uncertainty, by **Solea, et al., 2009.**

The most famous type of nonlinear control system and the oldest one is the discontinues control system, one of them specifically is the sliding mode control (SMC) system **Boiko, 2009.** In the study by **Utkin, et al., 2009,** it has been proven that the SMC system is efficient for controlling complex plants that have a high order of nonlinearity dynamics, that are operating with the existence of disturbance and uncertainty of system parameters. The variables of the sliding mode should be designed accurately, and this leads to proper closed-loop system performance when the trajectories of the system lie on the sliding manifold. **Shtesse, et al., 2014,** showed that the main idea behind the SMC is to drive the system path to the selected sliding manifold with the aid of control, and keep the motion on the manifold, which means utilizing the main characteristics of the sliding mode that is its insensitivity to internal and external perturbations matched by the control, maximum accuracy, and finite time convergence of the sliding variables to zero. SMC system has an advantage that its robustness against the matched disturbances, and also ease of implementation. However it has a disadvantage that the chattering behavior that appears for several reasons like the non-ideal switching process; as shown by **Al-samarraie, 2013.** One of the simplest methods is to attenuate the chattering behavior, as presented by **Slotine, 1983,** is by replacing the discontinuous signum function by a continuous function, like the saturation function.

The purpose of the adaptive controller, as shown by **Lavretsky and Wise, 2013,** is to perform real-time estimation to the process of uncertainty then will produce a control input to estimate and reduce the unwanted deviation from the expected closed-loop system performance**.** The adaptive sliding mode control (ASMC) system consists of a sliding mode controller and an adaptive controller. With the use of a sliding mode control system, the control effort can be reduced by selecting a proper control gain related to the change of system parameters and uncertainty. An ASMC methodology that guarantees a real sliding mode only was proposed by **Plestan, et al., 2010.** The sliding mode with gain-adaptation has been established without a priori knowing uncertainties/perturbations bounds while both the adaptive-gain values are not overestimated.

Based on the proposed work by **Plestan, et al., 2010,** the ASMC methodology is applied, in the present work, to design a robust controller to the nonholonomic mobile platform. The control design uses the concept of partial feedback linearization, after considering a certain point at the platform as the desired outputs; where the system dynamics concerning these outputs is minimum phase. The dynamical model to a mobile platform is derived here utilizing the Euler-Lagrange equation, where the nonholonomic constraints are considered through the Lagrange multiplier.

2. MATHEMATICAL MODEL FOR MOBILE PLATFORM

The mobile platform under study is made of a rigid cart driven by two DC motors, which is described by four degrees of freedom. **Fig. 1** shows the mobile platform with two wheels; where $\boldsymbol{r}$ is the radius of the mobile wheel (in $\boldsymbol{m}$), $\boldsymbol{2b}$ is the distance between the two wheels (in $\boldsymbol{m}$), $\boldsymbol{P_o}$ is the location of the point in between the wheels, $\boldsymbol{P_c}$ is a point on the platform, which is $\boldsymbol{d}$ distance (called look-ahead distance) from $\boldsymbol{P_o}$.

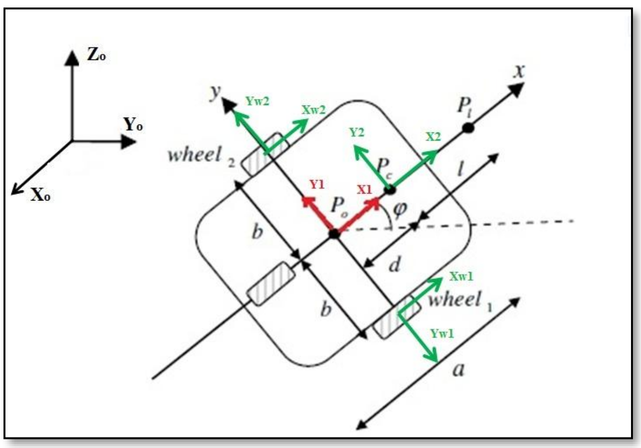

Figure 1. Mobile platform.

2.1. Kinematic Model of the Mobile Platform

Kinematics means a representation of the platform mathematically (displacement, velocity, and acceleration) with respect to a reference frame without any forces affecting its motion and velocity**,** as shown by **Siciliano, et al., 2009.** In this paper, the kinematic relations were evaluated with respect to inertial or reference coordinates. The configuration variables of the system are defined as $[x, y, \varphi, \theta_1, \theta_2]$, were x and y are the platform position in the horizontal plane (midway between the two wheels), φ is the rotation angle of the platform, the chassis, which is measured from the inertial x-axis, and finally$\theta_1$and θ_2 are the rotational angles of the first and the second wheel respectively. The nonlinear dynamics of chassis is coupled with the dynamics of the wheels. The body of the chassis is assumed to be symmetric and no slip condition is required. X_o and Y_o are the inertial reference frame, the frame X_{po} and Y_{po} are attached frame to the middle point between two wheels of mobile platform, and d is the distance from the origin of (X_{po}, Y_{po}) to (X_{pc}, Y_{pc}) on the X_{po}-axis as shown in **Fig. 1**.

2.1.1 Homogenous Transformation

The homogeneous transformation matrices which were presented by **Spong, et al., 2006,** was used in this paper to derive the kinematic equation of the wheels and chassis. The mobile platform is represented by five configuration variables; which are $[\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\varphi}, \boldsymbol{\theta_1}, \boldsymbol{\theta_2}]$, as mentioned above.

2 .2 Holonomic and Non-Holonomic Constraints

In dynamics, constraints on position and velocity of the system are introduced instead of unknown forces to describe the observed motions**, Cushman, et al., 2010.**

Holonomic constraints are typically introduced by mechanical interactions among the different parts of the system. The holonomic constraints are integrable equations, and hence can be represented as:

$$h_i(q) = 0,\ i = 1,2, \dots \dots \dots , m < n \tag{1}$$

where the function $h_i(q)$ is assumed to be smooth and independent.The system whose all constraints are holonomic is called a holonomic system. The effect of the holonomic constraints is to confine the attainable system configurations to $(n - m)$ dimensional smooth as showed by **De Luca and Oriolo, 1995**. In nonholonomic constraints, the system expression involves generalized coordinates and velocity in the form:

$$A_i(q, \dot{q})\ =\ 0, i =\ 1, \dots, k < n \tag{2}$$

They are referred to as kinematic constraints. The nonholonomic constraints will limit the admissible motions of the system by restricting the set of generalized velocity that can be attained at a given configuration. In mechanics, such constraints are usually encountered in the Pfaffian form **De Luca and Oriolo, 1995**:

$$a_i^T(\dot{q})q =\ 0, i = 1, \dots, k < n$$

These equations are nonintegrable, that is the equations cannot be put in the form given by Eq. (1). If all the system constraints are nonholonomic, it is called a non-holonomic system.

For the mobile platform, there are two types of constraints; holonomic and nonholonomic, which are presented in the following items.

2.2.1 Holonomic Constrain Equation for a Mobile Platform

There is only one holonomic constraint for the mobile platform. It is given by:

$$\dot{\varphi} = (v_1 -\ v_2)/2b = r(\ \dot{\theta}_1 - \dot{\theta}_2)/2b \tag{3}$$

By integrating Eq. (3), the holonomic constraint on the mobile platform was obtained as follows:

$$\varphi = C(\theta_1 -\ \theta_2),\ C = r/2b \tag{4}$$

where $v_1 = r\dot{\theta}_1$, and $v_2 = r\dot{\theta}_2$ are the linear velocities of the first and second wheels respectively, φ and $\dot{\varphi}$ are the rotational angel and the angular velocity of chassis respectively, finally, r is the radius of wheels and $2b$ is the distance between two wheels. As a result φ is expressed in terms of θ_1 and θ_2, and the number of degrees of freedom (DOF), that expresses the mobile platform system, becomes four.

2.2.2 Nonholonomic Constraints equation

Three nonholonomic constraints are imposed on the motion of the mobile platform. These constraints are given as follows:

2.2.2.1 The lateral constraint:

For the mobile platform, the velocity is equal to zero in the lateral direction. This represents the nonholonomic constraint, which was derived here as follows; the velocity of $\boldsymbol{P_o}$is

$$v_{p_\circ} = \begin{bmatrix} vx_\circ \\ vy_\circ \end{bmatrix} = \begin{bmatrix} C\varphi & -S\varphi \\ S\varphi & C\varphi \end{bmatrix} \begin{bmatrix} v\grave{x} \\ v\grave{y} \end{bmatrix}$$

From which the tangential and normal velocities with respect to the wheels are obtained as

$$\begin{bmatrix} v\grave{x} \\ v\grave{y} \end{bmatrix} = \begin{bmatrix} C\varphi & -S\varphi \\ S\varphi & C\varphi \end{bmatrix}^{-1} \begin{bmatrix} vx_\circ \\ vy_\circ \end{bmatrix} = \begin{bmatrix} C\varphi & S\varphi \\ -S\varphi & C\varphi \end{bmatrix} \begin{bmatrix} vx_\circ \\ vy_\circ \end{bmatrix} = \begin{cases} \dot{x}\cos\varphi + y^{\cdot}\sin\varphi \\ -\dot{x}\sin\varphi + \dot{y}\cos\varphi \end{cases}$$

As can be observed from **Fig. 2**, the mobile platform lateral velocity$v\grave{y}$ is equal to zero. That means the first nonholonomic constraint is:

$$-\dot{x}\sin\varphi + \dot{y}\cos\varphi = 0 \tag{5}$$

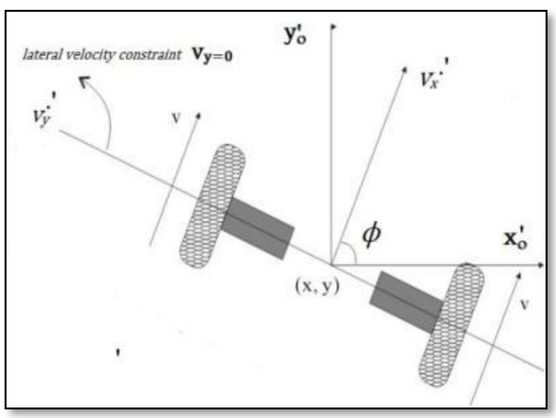

Figure 2. Lateral constraint.

2.2.2.2. No slip condition constraint

No slip condition was brought to light where the velocity of the wheels of the point touch to the ground is equal to zero. This instant of zero velocity represents the second and third nonholonomic constraints. It is derived as follows (**Fig. 3**); the first wheel velocity vector at the point touch to the ground is obtained as:

$$\grave{v}_{w1} = \begin{bmatrix} \grave{v}_{w1x} \\ \grave{v}_{w1y} \end{bmatrix} = \begin{bmatrix} C\varphi & S\varphi \\ -S\varphi & C\varphi \end{bmatrix} \begin{bmatrix} v^\circ_{w1} \\ v^\circ_{w1} \end{bmatrix} = \begin{bmatrix} C\varphi & S\varphi \\ -S\varphi & C\varphi \end{bmatrix} \begin{bmatrix} \dot{x} + bC\varphi\dot{\varphi} \\ \dot{y} + bS\varphi\dot{\varphi} \end{bmatrix}$$

The no-slip for the first wheel means $\dot{v}_{w1x} = 0$, hence the second nonholonomic constraint is obtained as:

$$\dot{x}\,\cos\varphi + \dot{y}\,\sin\varphi + b\dot{\varphi} = 0 \tag{6}$$

In the same manner, the third nonholonomic constraint due to no-slip condition on the second wheel is obtained as;

$$\dot{x}\cos\varphi + \dot{y}\,\sin\varphi - b\dot{\varphi} = 0 \tag{7}$$

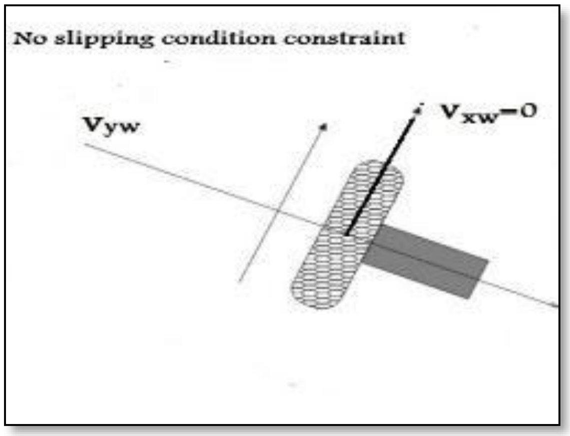

Figure 3. No slipping condition constraint.

In pfaffian form, the nonholonomic constraint matrix is given by:

$$A(q)\dot{q} = \begin{bmatrix} -\sin\varphi & \cos\varphi & 0 & 0 & 0 \\ \cos\varphi & \sin\varphi & b & -r & 0 \\ \cos\varphi & \sin\varphi & -b & 0 & -r \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Due to the holonomic constraint (Eq. 4), the Pfaffian form is reduced to:

$$A(q)\dot{q} = \begin{bmatrix} -\sin\varphi & \cos\varphi & 0 & 0 \\ \cos\varphi & \sin\varphi & -r/2 & -r/2 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{8}$$

3. DYNAMICS OF PLATFORM

In this section, the dynamical model of the platform is introduced with considering the holonomic and nonholonomic constraints that were derived in the previous section; where the Euler-Lagrange equation is utilized here to derive the mobile platform mathematical model**.**

Lagrangian L is the difference between the system kinetic and potential energy

$$L(q,\dot{q}) = T(q,\dot{q}) - U(q) = 1/2(\dot{q}^T M(q)\dot{q}) - U(q) \tag{9}$$

Where M(q) is the positive definite inertia and a mass matrix of the system, and $\mathrm{U(q)}$ is the potential energy of the platform. For the mobile platform, the equation of motion, after applying the Euler-Lagrange equation, is given by;

$$\mathrm{M(q)\ddot{q} + D(q,\dot{q})\dot{q} + G(q) = A^T(q)\,\lambda + E(q)u} \tag{10}$$

where the mass matrix M(q) for the platform is given by, Appendix (A);

$$\mathrm{M(q)} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

$\mathrm{D(q,\dot{q})\dot{q}}$, and $\mathrm{G(q)}$ is the carioles matrix and the potential energy respectively. Both $\mathrm{D(q,\dot{q})\dot{q}}$, and $\mathrm{G(q)}$ are equal to zero for the platform system model.

As can be noted from Eq. (8), the nonholonomic constraint matrix $\mathrm{A(q)}$ is not of full rank $\forall q$. So there exists a null space associated to matrix A(q), i.e., $Null(\mathrm{A(q)}) \neq \emptyset$. The null space for the mobile platform is spanned by the rows of the following matrix;

$$\begin{bmatrix} r/2\cos\varphi & r/2\sin\varphi & 1 & 0 \\ r/2\cos\varphi & r/2\sin\varphi & 0 & 1 \end{bmatrix} = \mathrm{N^T(q)} \tag{11}$$

As a result of the columns of matrix N(q), the basis for the null space of A(q), so $\mathrm{N^T(q)A(q) = 0}$.

The term $\mathrm{A^T(q)\,\lambda}$, in Eq. (10), is the vector of constraint forces; which is altered by the magnitude of the Lagrange multiplier λ. This term represents the effect of the nonholonomic constraint on the system dynamics. To eliminate the unknown Lagrange multiplier term from the dynamical model, $\mathrm{N^T(q)}$ is multiplied by Eq. (10), as in the following;

$$\begin{aligned} &\mathrm{N^T(q)\,\{M(q)\ddot{q} + D(q,\dot{q})\dot{q} + G(q) = A^T(q)\,\lambda + E(q)u\}} \\ &\Longrightarrow \mathrm{N^T(q)M(q)\ddot{q} = N^T(q)E(q)u} \end{aligned} \tag{12}$$

To this end the following change of variable is considered;

$$\dot{q} = N(q)\rho \tag{13}$$

whereρ $\in R^2$ is a new vector state, which lies in the null space of A(q). The total platform system state, after considering the nonholonomic constraints, are given by; $q = [x \quad y \quad \theta_1 \quad \theta_1]^T$and ρ. Rename the state as; $x_1 = x, x_2 = y, x_3 = \theta_1, x_4 = \theta_2$and $\rho = [x_5 \quad x_6]^T$, i.e.,

$$\begin{bmatrix} q \\ \rho \end{bmatrix} = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \tag{14}$$

which represents the total mobile platform state after considering the holonomic and nonholonomic constraints. With respect to the new state (Eq. (14)), the velocity vector for the nonholonomic platform model becomes;

$$\begin{bmatrix}\dot{q}\\ \dot{\rho}\end{bmatrix}=\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix}=\begin{bmatrix}N(q)[x_5 \quad x_6]^T\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} \tag{15}$$

Where $\dot{\rho}$ is computed by substituting Eq. (13) in Eq. (12)

$$\ddot{q}=\frac{d}{dt}(N(q)\rho)=\left\{\frac{d}{dt}N(q)\right\}\rho+N(q)\frac{d}{dt}\rho=\dot{N}(q)\rho+N(q)\dot{\rho}$$

$$\Longrightarrow N^T(q)M(q)\dot{N}(q)\rho+N^T(q)M(q)N(q)\dot{\rho}=N^T(q)E(q)u$$

Then$\dot{\rho}$ is obtained as

$$\dot{\rho}=[N^T(q)M(q)N(q)]^{-1}\{-N^T(q)M(q)\dot{N}(q)\rho+N^T(q)E(q)u\} \tag{16}$$

where$N^T(q)M(q)N(q)$ is an invertable $2x2$ matrix. Using Eq. (16), the nonholonomic platform dynamical model is given by;

$$\begin{bmatrix}\dot{q}\\ \dot{\rho}\end{bmatrix}=\begin{bmatrix}N(q)[x_5 \quad x_6]^T\\ -[N^T(q)M(q)N(q)]^{-1}N^T(q)M(q)\dot{N}(q)\rho+[N^T(q)M(q)N(q)]^{-1}N^T(q)E(q)u\end{bmatrix}$$

$$=\begin{bmatrix}f_1(q,\rho)\\ f_2(q,\rho)+g(q,\rho)u\end{bmatrix} \tag{17}$$

Where $f_1(q,\rho)=N(q)[x_5 \quad x_6]^T,\ f_2(q,\rho)=-[N^T(q)M(q)N(q)]^{-1}N^T(q)M(q)\dot{N}(q)\rho$ and $g(q,\rho)=[N^T(q)M(q)N(q)]^{-1}N^T(q)E(q)$.

4.PARTIAL FEEDBACK LINEARIZATION

Feedback linearization is a method that is used to transform the nonlinear system into an equivalent linear system. It leads to full-state linearization, where the state equation is completely linearized. On the other hand, a partial feedback linearization is an input-output linearization, where the input-output map is linearized, while the state equation may be only partially linearized, **Khalil, 2002**. Additionally, **Isidori, 2000,** showed that the system must be a minimum phase to guarantee the stability of the control system when designed based on the input-output linearization.

In this work, the mobile platform model is partially linearized considering the position of the point P_c **(Fig. 1**) with respect to the wheels torques inputs. It was shown by **Yun, et al., 1992**, that the mobile platform is input-output linearizable at P_c. That also means, the mobile platform is minimum phase when controlling it via static state feedback through the point P_c**.**
The position functions of mobile platform throw $\boldsymbol{P_c}$ point is obtained by using the homogeneous transformation matrices method as demonstrated below.

$$T_{pc}^{\circ} = T_{Po}^{\circ} T_{Pc}^{po} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & x \\ \sin\varphi & \cos\varphi & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & x + d\cos\varphi \\ \sin\varphi & \cos\varphi & 0 & y + d\sin\varphi \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow P_{pc}^{0} = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 & x + d\cos\varphi \\ \sin\varphi & \cos\varphi & 0 & y + d\sin\varphi \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x + d\cos\varphi \\ y + d\sin\varphi \\ 0 \\ 1 \end{bmatrix}$$

where T_{pc}° and P_{pc}^{0} are the homogeneous transformation matrix and the position of $\boldsymbol{P_c}$ with respect to the inertial frame 0. From P_{pc}^{0}, the required outputs (y_1 and y_2) are obtained as;

$$\left.\begin{aligned} y_1 &= p_{pcx}^{0} = x + d\cos(\varphi) \\ y_2 &= p_{pcy}^{0} = y + d\sin(\varphi) \end{aligned}\right\} \quad (18)$$

The time derivative of the outputs is evaluated using Eqs. (16) and (4);

$$\left.\begin{aligned} \dot{y}_1 &= [(r/2)\cos\varphi - cd\sin\varphi \quad (r/2)\cos\varphi + cd\sin\varphi] \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \\ \dot{y}_2 &= [(r/2)\sin\varphi + cd\cos\varphi \quad (r/2)\sin\varphi - cd\cos\varphi] \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} \end{aligned}\right\} \quad (19)$$

After that $\ddot{y}_1$ and $\ddot{y}_2$ are obtained as

$$\left.\begin{aligned} \ddot{y}_1 &= f_1(x) + g_1(x)u \\ \ddot{y}_2 &= f_2(x) + g_2(x)u \end{aligned}\right\} \quad (20)$$

where

$$f_1(x) = \left[-\frac{r}{2\sin(\varphi)\dot{\varphi}} \quad -\frac{r}{2}\sin(\varphi)\dot{\varphi}\right] \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} +$$

$$[r/2\cos(\varphi) - cd\sin(\varphi) \quad r/2\cos(\varphi) + cd\sin(\varphi)][N^T(q)M(q)N(q)]^{-1} \left[- N^T(q)M(q)\dot{N}(q) \begin{bmatrix} x_5 \\ x_6 \end{bmatrix}\right]$$

$$f_2(x) = [\, r/2\cos(\varphi)\,\dot{\varphi} \quad r/2\cos(\varphi)\,\dot{\varphi}] \begin{bmatrix} x_5 \\ x_6 \end{bmatrix} - d\sin(\varphi)\dot{\varphi}^2 +$$

$$[r/2\sin(\varphi) + cd\cos(\varphi) \quad r/2\sin(\varphi) - cd\cos(\varphi)][N^T(q)M(q)N(q)]^{-1} \left[- N^T(q)M(q)\dot{N}(q) \begin{bmatrix} x_5 \\ x_6 \end{bmatrix}\right]$$

$$g_1(x) = [r/2\cos(\varphi) - cd\sin(\varphi) \quad r/2\cos(\varphi) + cd\sin(\varphi)\,][(N(q)t\big(M(q)N(q)\big)^{-1}]$$

$$g_2(x) = [r/2\cos(\varphi) + cd\sin(\varphi)\frac{r}{2}\sin(\varphi) - cd\cos(\varphi)\,]\;[(N(q)t\big(M(q)N(q)\big)^{-1}]$$

To this end, the outputs dynamics are rewritten in the error function form; that is by defining $e_1 = y_1 - y_{1d}$and $e_2 = y_2 - y_{2d}$, where y_{1d} and y_{2d} are the desired position for y_1 and y_2Respectively. Accordingly, $\dot{e}_1$ and $\dot{e}_2$ are computed as follows;

$$\left.\begin{aligned}\dot{e}_1 &= [(r/2)\cos\varphi - cd\sin\varphi \quad (r/2)\cos\varphi + cd\sin\varphi]\begin{bmatrix}x_5\\x_6\end{bmatrix} - \dot{y}_{1d}\\ \dot{e}_2 &= [(r/2)\sin\varphi + cd\cos\varphi \quad (r/2)\sin\varphi - cd\cos\varphi]\begin{bmatrix}x_5\\x_6\end{bmatrix} - \dot{y}_{2d}\end{aligned}\right\} \tag{21}$$

And $\ddot{e}_1$ and $\ddot{e}_2$ are

$$\ddot{e} = \begin{bmatrix}\ddot{e}_1\\ \ddot{e}_2\end{bmatrix} = \begin{bmatrix}F_1(x)\\F_2(x)\end{bmatrix} + \begin{bmatrix}G_1(x)\\G_2(x)\end{bmatrix}u = F(x) + G(x)u \tag{22}$$

where $F_1(x) = f_1(x) - \ddot{y}_{1d}$, $F_2(x) = f_2(x) - \ddot{y}_{2d}$, $G_1(x) = g_1(x)$ and $G_2(x) = g_2(x)$. Considering the uncertainty in the system model, the error function dynamics can be put as;

$$\ddot{e} = F_o(x) + G_o(x)u + \delta(x,u) \tag{23}$$

where $F_o(x)$ and $G_o(x)$ are $F(x)$ and $G(x)$ but with nominal parameters value, while $\delta(x,u)$ is the perturbation term due to system model uncertainty. Mathematically, $\delta(x,u)$ is the difference between the right-hand side of Eq. (22) and $F_o(x) + G_o(x)u$, i.e.,

$$\left.\begin{aligned}\delta(x,u) &= F(x) + G(x)u + F_r - (F_o(x) + G_o(x)u) = \Delta F(x) + \Delta G(x)u\\ &= \Delta F(x) + \Delta G(x)u + F_r\end{aligned}\right\} \tag{24}$$

Here F_r is the friction torques, which is given by;

$$F_r = -N^T * \left[F_v * \begin{bmatrix}x_5\\x_6\end{bmatrix} + F_s * sign\begin{bmatrix}x_5\\x_6\end{bmatrix}\right] \tag{25}$$

5. SMC

The SMC is a specific type of variable structure control (VSC) which is characterized by a suite of feedback control laws and decision rule. It is termed the switching function utilization. This natural idea began in the soviet union in the late 1950s.

SMC is designed to bring the state to the sliding manifold ($s = 0$) and then constrain the system state within the neighborhood of the sliding manifold. There are two main advantages in this method, the first one is that the dynamic system behavior may be designed by the specific selection of switching function, and the second one is that the closed loop response become completely insensitive to a particular class of uncertainty. The later invariance property makes the methodology a suitable candidate for robust control. On the other hand, the main SMC disadvantage is the system chattering. This behavior is the result of the discontinuity in SMC formula, and is related to the controller gain value.

The sliding mode design consists of two parts, one involves the design of switching function, hence the sliding motion meets the design specification, and the other is related to the selection of the control low which will make the switching manifold attractive to the system state, **Edwards and Spurgeon, 1998**. To improve the SMC performance, the SMC gain will be evaluated via adaptive law as presented in the next subsection.

The main objective of the present paper is to design a nonlinear controller based on the ASMC theory to enforce the point $\boldsymbol{P_c}$ on the platform to follow the desired trajectory in spite of system dynamic model uncertainty (the perturbation term $\delta(x, u)$). As in the conventional SMC, the first step is the selection of the switching function $s = [s_1 \quad s_2]^T$. Here s is selected as follows;

$$s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} + \Lambda \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \dot{e} + \Lambda e \tag{26}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix},\ \lambda_1, \lambda_2 > 0 \tag{27}$$

where $\lambda_1, \lambda_2 > 0$is the required condition that guarantee the control system stability during sliding motion.

In the second step, the SMC is determined. In this step, the time derivative of s is required, which based on the derived error function dynamics (Eq. (23))

$$\begin{aligned} \dot{s} &= \begin{bmatrix} \dot{s}_1 \\ \dot{s}_2 \end{bmatrix} = \begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{bmatrix} + \Lambda \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \ddot{e} + \Lambda \dot{e} \\ &= F_o(x) + G_o(x)u + \delta(x, u) + \Lambda \dot{e} \end{aligned} \tag{28}$$

In the second step, the control input vector is given by:

$$u = G_o^{-1}(x)(u_o + \ u_s),\ |G_o(x)| \neq 0\ \forall x \tag{29}$$

where u_o and u_s are the nominal and discontinuous control vector respectively. Accordingly, $\dot{s}$ becomes;

$$\dot{s} = F_o(x) + u_o + u_s + \delta(x, u) + \Lambda \dot{e} \tag{30}$$

The nominal control u_o is selected here as

$$u_o = -F_o(x) - \Lambda \dot{e} \tag{31}$$

As a result, $\dot{s}$ in terms of the discontinuous control vector becomes

$$\dot{s} = u_s + \delta(x, u) \tag{32}$$

The u_s control term will be designed based on the ASMC in the next subsection.

5.1 Adaptive SMC

As mentioned earlier, the ASMC which was designed in the present work is based on the work proposed by **Plestan, et al., 2010**. In the present work, the discontinuous control $\boldsymbol{u_s}$with the adapted gain $\boldsymbol{K(t)}$ is given in the following form

$$\boldsymbol{u_s(s\,, t)\ =\ -K(t)\ sign(s(x(t)\,, t))} \tag{33}$$

With $K(t)$ satisfies

$$\dot{K}(t) = \begin{cases} \gamma|s(x(t),t)| * sign(|s(x(t),t)| - \varepsilon) & if K > \mu \\ 0 & if\ K \le \mu \end{cases} \quad (34)$$

Equation (34) is the adaptation law for $K(t)$, while $s\,(x(t),t)$ is the switching function and $\gamma, \varepsilon, \mu > 0$ are the adaptation law parameters with a significantly positive small value of μ. μ is incorporated so that only positive K values can be achieved. Once there is an establishment of a sliding motion with respect to $s(x(t),t)$, the suggested gain adaptation law (34) declines the gain K while $|s\,(x(t)\,,t)| < \varepsilon$. Similarly, gain K will be maintained at the least level that will allow the stabilization of a given accuracy of s. This adaptation law allows attaining an adequate gain with respect to the magnitude of the uncertainties and perturbations, **Plestan, et al., 2010**.

5.1.2 The discontinuous control law for the mobile platform

The proposed discontinuous control term and the adaptation law for the mobile platform are as follows;

$$u_{si} = -k_i(t) sign(s_i)\ i = 1,2 \quad (35)$$

$$\dot{\mu}_i(t) = \gamma_i * |s_i| * sign(|s_i| - \epsilon_i) \quad (36)$$

where ϵ_i is a small positive constant. The adapted gain $k_i(t)$ is proposed to be evaluated according to the following rules:

$$k_i(t) = \begin{pmatrix} \mu_i & if K_{imin} < \mu_i < K_{imax} \\ K_{imin} & if \mu_i \le K_{imin} \\ K_{imax} & if \mu_i \ge K_{imax} \end{pmatrix} \quad (37)$$

where $\gamma_i > 0$, $\epsilon_i > 0$, while K_{imin} and K_{imax} are the minimum and maximum possible value of $k_i(t)$ respectively.

6. SIMULATION RESULTS AND DISCUSSION

In this work, the simulation results of mobile platform trajectory tracking were obtained using the Matlab software. Two desired trajectories were utilized as a reference trajectory; these are infinity shape and circle shape trajectory. Additionally, the system was considered uncertain, due to the parameters variation, with the existence of the friction torques acting as external disturbances.

The mobile platform nominal parameters are listed in **Table 1**, while **Table 2** presents the maximum value of chassis mass and the moment of inertia of chassis in z and y directions due to uncertainty. This information was used in the numerical simulations presented below. Additionally, **Table 3** gives control parameters, which have been used to obtain the simulations results for different reference trajectories.

Table 1. Mobile platform nominal parameters.

parameter	Definition	Values	Units
R	the radius of each driving wheel	0.1	m
B	the distance between the driving wheels and the axis of symmetry	0.15	m
D	Distance between point $\boldsymbol{p}_{\circ}$ and point $\boldsymbol{p_c}$ on the plat form surface	0.5	m
$\mathbf{I_{xx}}$	the moment for inertia of chassis in the x-direction	0	m
$\mathbf{I_{yy}}$	the moment for inertia of chassis in the y-direction	0.139	Kg.$\mathbf{m^2}$
$\mathbf{I_{zzc}}$	moment of inertia of chassis in the z-direction	6.609	Kg.$\mathbf{m^2}$
$\mathbf{I_{zzw}}$	moment of inertia of wheels in the z-direction	0.010	Kg.$\mathbf{m^2}$
$\mathbf{m_c}$	chassis mass	94	kg
$\mathbf{m_w}$	Wheel mass	5	kg

Table 2. Mobile platform parameter with 100% uncertainty used for (ASMC) simulation.

parameter	Definition	value	unit
$\mathrm{m_c}$	The maximum value of chassis mass	188	Kg.
$\mathrm{I_{zzc}}$	Maximum moment of inertia of chassis in the z-direction	13.218	Kg.$\mathrm{m^2}$
$\mathrm{I_{yy}}$	Maximum moment for inertia of chassis in the y-direction	0.278	Kg.$\mathrm{m^2}$

The results of applying the ASMC for the mobile platform where the desired trajectory is the infinity shape are presented in **Figs. 4 to 7**. The ASMC system performance is well clarified in **Fig. 4**, where the actuation torques force the mobile platform to follow the infinity shape. This result is also clear in **Fig. 5**, where it is required less than 9 sec. for the mobile platform to follow the infinity shape. Unlike the traditional SMC, the discontinuous gains k_1 and k_2 are adapted to ensure the existence of sliding motion; also, the adaptation process do not need any information about the bound of system parameters or disturbances due to friction. The plot of k_1 and k_2 is shown in **Fig. 6,** while **Fig. 7** shows that the sliding variables s_1 and s_2 were effectively regulated to the sliding manifold in less than 1 sec., despite of the uncertainty in system model. Eventually, the results proved that unlike traditional SMC, the ASMC was able to adapt each discontinuous gain to direct its own sliding variable to zero level, irrespective to the existence of coupling between sliding variables dynamic. The hierarchy control was one of the pioneering method, which proposed by **Utkin, 1992** to solve the coupling problem in sliding variable dynamics; however, the result was a high gain values, which will induce a severe chattering in system response especially when applied to mechanical systems. For this reason, the proposed work here can be regarded as the solution for designing a SMC for multi-input multi-output control systems, where the chattering problem is eliminated or greatly attenuated, with preserving the main SMC features the following **Table 3** contain control parameters.

Table 3. Control adaptation law parameters and control gain values.

parameter	Definition	Value
γ_1	Positive number for first adaptation gain	3
γ_2	Positive number for second adaptation gain	4
$\varepsilon_{1,2}$	Very small positive number for first and second adaptation law	0.02
k_1	Initial value for first control gain	15
k_2	Initial value for second control gain	20
$k_{1\,min}$	Minimum value for first control gain	10
$k_{1\,max}$	Maximum value for first control gain	20
$k_{2\,min}$	Minimum value for second control gain	20
$k_{2\,max}$	Maximum value for second control gain	25

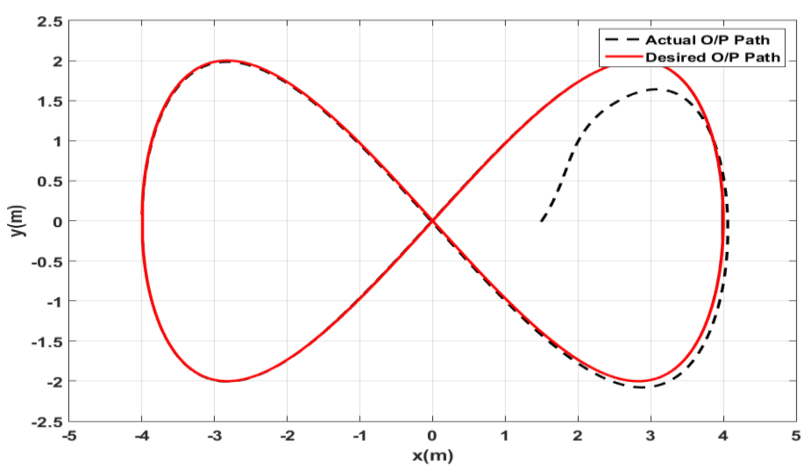

Figure 4. Infinity shape trajectory tracking.

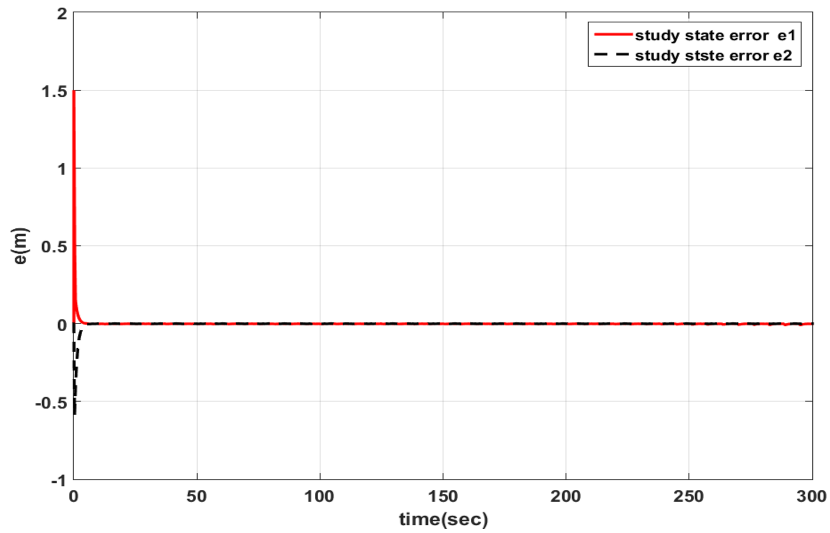

Figure 5. Study state error e_1, e_2.

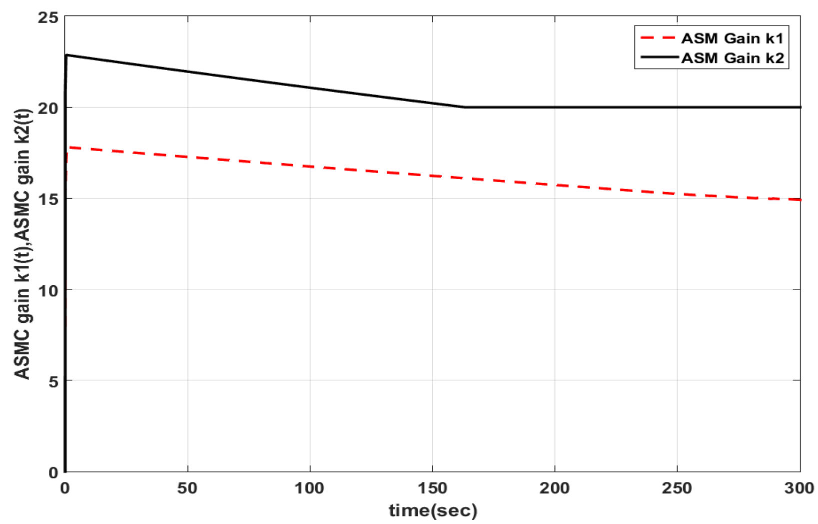

Figure 6. The adapted gains k_1, and k_2.

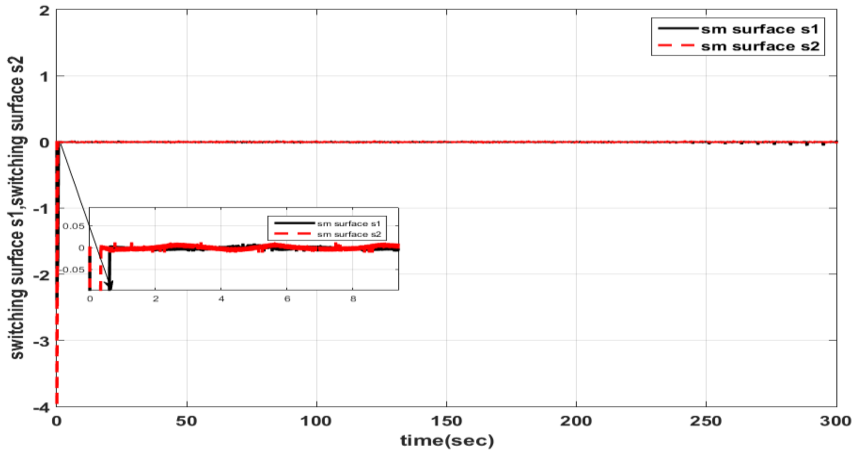

Figure 7. Sliding variables s_1, s_2.

The proposed ASMC was also applied to direct the mobile platform to follow a circular trajectory shape. **Figures 8, 9, 10** and **11** show the mobile platform trajectory, the error functions, the adapted gains, and the sliding variable respectively for the circle desired shapes, where the ability and effectiveness of the ASMC can be observed from these figures.

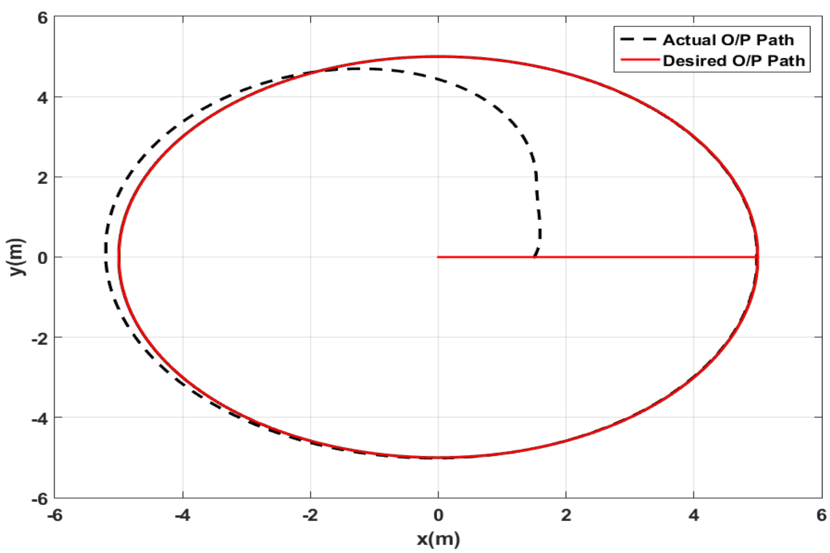

Figure 8. Circle trajectory tracking.

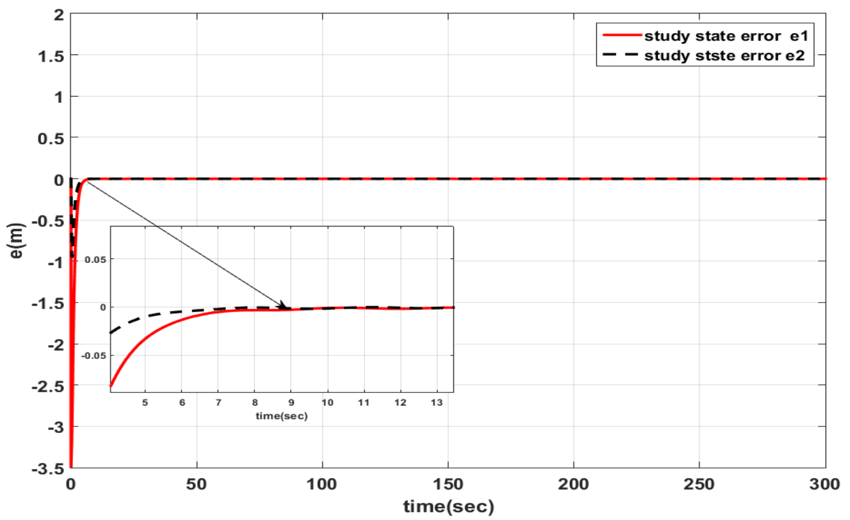

Figure 9. Study state errors e_1, and e_2.

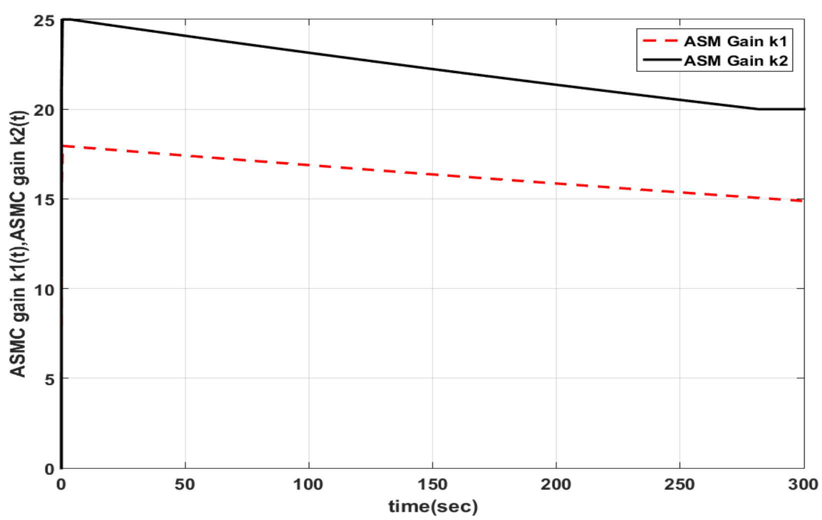

Figure 10. The adapted gains k_1, and k_2.

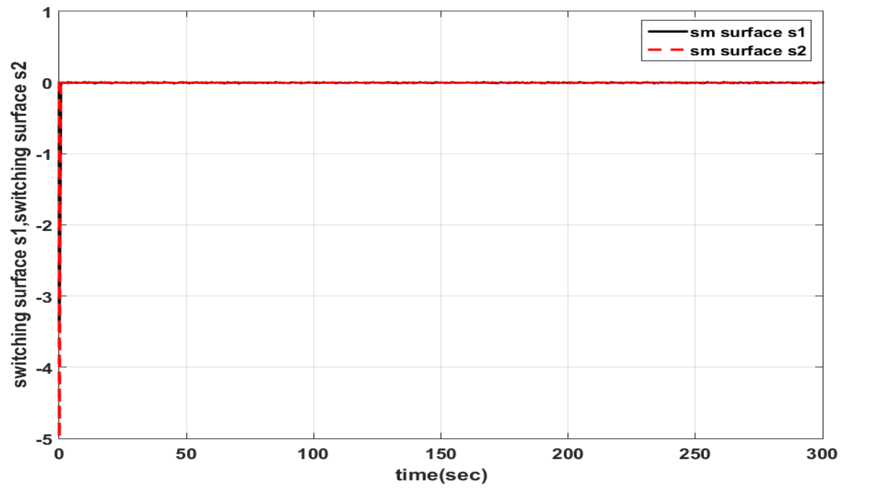

Figure 11. Sliding variables s_1, and s_2.

7. CONCLUSIONS

In this work, an ASMC was designed to control the nonholonomic mobile platform trajectory tracking. The mathematical model was successfully derived considering one holonomic and two nonholonomic constraints. Since the mobile platform at $\boldsymbol{P_c}$, is input-output linearizable, the partial feedback linearization was utilized to derive the error function model. As a result, the obtained error function model is 2DOF system; this enabled to design a fully actuated control system to the mobile platform, since there exist two actuators at the wheels. After that, the ASMC was used to design a robust adaptive control system, which forced the mobile platform to follow the desired trajectory. Using numerical simulations to depict the dynamical behavior of the mobile platform with the presence of parameters uncertainty and friction torques, the results have shown good performance of the proposed ASMC; where the mobile platform followed different desired paths, like infinity and circle shapes.

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Appendix A: The mass matrix M(q)

The elements of the mass matrix M(q) was calculated using the Euler- Lagrange equation. As it is well known, finding the kinetic and potential energies is required for deriving the mass matrix. The mass matrix is presented as

$$M(q) = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}$$

where $m_{11} = (m_c + 2m_w)$, $m_{12} = m_{13} = m_{14} = m_{21} = 0$, $m_{22} = (m_c + 2m_{\,w})$ $m_{23} = m_{24} = m_{31} = m_{32} = 0$, and

$$m_{33} = \mathrm{I_{zzc}}c^2 + \ + 2\mathrm{m_w}b^2c^2 + \mathrm{I_{zzw}}c^2 + \mathrm{I_{zw}}c^2 + \mathrm{I'_{yy}}$$

$$m_{34} = -I_{zz}c^2 - 2m_W b^2c^2 - 2I_{\,ZW}c^2$$

$$m_{41} = m_{42} = 0, m_{43} = -I_{zz}c^2 - 2m_W b^2c^2 - 2I_{\,ZW}c^2$$

$$m_{44} = I_{zz}c^2 + 2m_w b^2c^2 + 2I_{zw}c^2 + \mathrm{I_{yy}}$$