Adaptive Sliding Mode Controller for Servo Actuator System with Friction

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ABSTRACT

This paper addresses the use of adaptive sliding mode control for the servo actuator system with friction. The adaptive sliding mode control has several advantages over traditional sliding mode control method. Firstly, the magnitude of control effort is reduced to the minimal admissible level defined by the conditions for the sliding mode to exist. Secondly, the upper bounds of uncertainties are not required to be known in advance. Therefore, adaptive sliding mode control method can be effectively implemented. The numerical simulation via MATLAB 2014a for servo actuator system with friction is investigated to confirm the effectiveness of the proposed robust adaptive sliding mode control scheme. The results clarify, after comparing it with the results obtained by using classical sliding mode control, that the control efforts are reduced and the chattering amplitude is attenuated with preserving main features of the classical sliding mode control.

Key words: sliding mode control, adaptive sliding mode control, low pass filter, servo actuator system.

تصميم مسيطر منزلق متكيف لمنظومة المحرك المؤازر بوجود الاحتكاك

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الخلاصة

يتناول هذا البحث استخدام مسيطر منزلق متكيف للسيطرة الرصينة على منظومة المحرك المؤازر بوجود الاحتكاك . المسيطر المنزلق المتكيف لديه العديد من المزايا بالاضافة الى مزايا المسيطر المنزلق التقليدي. اولاً، يتم تقليل جهد التحكم الى اقل حد ممكن والتي تحدد بتحقق ظروف الانزلاق. ثانياً، ليس مطلوباً ان تكون الحدود العليا للشكوك معروفة سلفاً. لذلك ، يمكن تطبيق الطريقه المقترحة بفعالية. بعد مقارنة المحاكات العددية بإستخدام برنامج ال MATLAB 2014a لمنظومة المحرك المؤازر بوجود الاحتكاك مع نتائج المسيطر المنزلق النمطي تأكد كفاءة الخطة المعروضة للمسيطر المنزلق المتكيف الرصين. وتوضح النتائج تخفيض جهد التحكم وتقليل الارتجاج مع الاحتفاظ بالملامح الرئيسية للمسيطر الانزلاقي التقليدي.

الكلمات الرئيسية : المسيطر المنزلق، المسيطر المنزلق المتكيف، مرشح المرور الضئيل، منظومة المحرك المؤازر.

1. INTRODUCTION

Sliding mode control (SMC) is an effective nonlinear robust control approach that can deal with system uncertainties. When the perturbations satisfy the matching condition the robustness of control system is very important because various uncertainties exist in practical applications. It has been widely studied and successfully used in practical applications such as pendulum systems, robot manipulators, power converters and motors**, Utkin, et al., 2009**. Therefore, developing simple and robust controllers for uncertain system is of much importance.

Adaptive controller is thus a controller that can adjust its behavior in response to make changes in the dynamics of the process and character of the disturbances**, Astrom,** and **Wittenmark, 2008.** The adaptive sliding mode control (ASMC) is the combination of adaptive control method and SMC approach. It is more flexible and convenient in controller design than SMC and also the control method system stability can be achieved with a smaller control effort than that in SMC.

The conventional SMC uses a control law with high control gains yielding the undesired chattering. While the control system is in the sliding mode the chattering phenomenon is the main drawback of the SMC which can damage actuators and systems. The boundary layer method is the first approach to reduce the chattering where many ways have proposed adequate controller gains tuning**, Slotine,** and **Sastry, 1983.** In the second approach a higher order sliding mode control is used to attenuate the chattering phenomenon**, Bartolini, et al., 2000** and**, Laghrouche, et al., 2007**. However, in these both control approaches; knowledge of parameters uncertainty bounds are required. In**, Chang, et al., 2002** the system performance is satisfactory when more than one parameter adaptation is used in the control law. In**, Yoon,** and **Trumper, 2014** a frequency domain measurement technique was suggested to identify the friction model, and the model is used to compensate the nonlinear friction in the systems. Many controllers based on fuzzy tools**, Munoz,** and **Sbarbaro, 2000** and**, Jiang, et al., 2015** have been published; but, these papers do not ensure the tracking performances. **Hall,** and **Shtessel, 2006** used sliding mode disturbance observer when they proposed a gain-adaptation algorithm. The required knowledge of parameters uncertainty bounds to design observer-based controller is the main obstacle in implementing the proposed adaptive controller.

The purpose of this paper is to design a SMC with parameter uncertainties and external disturbances without requiring knowing the bound of uncertainties. This can be done based on a gain adaptation law derived in reference**, Plestan, et al., 2010.**

The organization of the present paper is as follows. In section 2, the servo actuator system is described including the friction model. In section 3, the SMC is explained briefly while the ASMC algorithm is given in Sec. 4. The simulation of a servo actuator system with friction is illustrated in Sec. 5 and the conclusions are given in Sec. 6.

2. DC SERVO ACTUATOR SYSTEM WITH FRICTION: MATHEMATICAL MODEL DESCRIPTION

The mathematical model of the servo actuator system is represented by a second order dynamic system with friction present between two contacting surfaces **AlSamarraie, 2013**, i.e.,

$$J\ddot{x} = u - F - T_L \tag{1}$$

where J is the moment of inertia; $x, \dot{x}, \ddot{x}$ are the actuator position, velocity and acceleration, respectively; u is the control input torque; F is the friction torque; and T_L is the load torque.

The friction torque is represented by static friction a phenomenon which includes: Coulomb friction, Stiction friction, and the viscous friction, i.e., **Hélouvry, et al., 1994.**

$$F = \left\{F_s \exp\left(-\left(\frac{\dot{x}}{\dot{x_s}}\right)^2\right) + F_c\left(1 - \exp\left(-\left(\frac{\dot{x}}{\dot{x_s}}\right)^2\right)\right) + \sigma|\dot{x}|\right\} * sgn(\dot{x}) \tag{2}$$

where F_c is the Coulomb friction, F_s is the Stiction friction, $\dot{x_s}$ is the Stribeck velocity, and σ is the viscous friction coefficient.

By defining, $e_1 = x - x_d$ and $e_2 = \dot{x} - \dot{x}_d$, the system model in Eq. (1) can be written in a state space structure as:

$$\left.\begin{aligned} \dot{e}_1 &= e_1 \\ \dot{e}_2 &= \left(\frac{1}{J}\right)(u - F - T_l) - \ddot{x}_d \end{aligned}\right\} \tag{3}$$

While the schematic diagram for the closed loop position servo actuator model is shown in Fig. 1.

Now Eq. (3) may also be written in terms of nominal and perturbation terms as;

$$\left.\begin{aligned} \dot{e}_1 &= e_1 \\ \dot{e}_2 &= f_{2o}(e) + g_{2o}u + \delta(e,u) \end{aligned}\right\} \tag{4}$$

where

$$f_{2o}(e) = -\ddot{x}_d,\ \ g_{20} = \left(\frac{1}{J_o}\right),\ \delta(e,u) = \Delta\left(\frac{1}{J}\right)u - \left(\frac{1}{J}\right)(F + T_l) \text{ and } \Delta\left(\frac{1}{J}\right) = \frac{1}{J} - \frac{1}{J_o}$$

The reference signal is chosen to be differentiable function. The position, velocity and acceleration references are chosen as**, Xie, 2007.**

$$\left.\begin{aligned} x_d &= \frac{1}{16\pi}\sin(8\pi t) - \frac{1}{24\pi}\sin(12\pi t) \\ \dot{x}_d &= \sin(10\pi t) * \sin(2\pi t) \\ \ddot{x}_d &= 10\pi * \cos(10\pi t)\sin(2\pi t) - 2\pi * \sin(10\pi t)\cos(2\pi t) \end{aligned}\right\} \quad (5)$$

One of the uses of the following signal would be in the antenna and array positioning system**, Xie, 2007.**

3. SLIDING MODE CONTROL

The SMC includes a discontinuous control which is used to enforce the state trajectories of the system to some manifold. Sliding mode belongs to the motion in the manifold, limited by desired properties of reduced-order differential equations**, Utkin, 2013**. SMC is an extremely fashionable strategy for management of nonlinear uncertain systems, with a really giant frame of applications fields**, Slotine,** and **Sastry, 1983**. Attributable to the utilization of the discontinuous function, its key features are the robustness of closed loop system and the finite time affluence. The main obstacles of pure SMC in practical applications are firstly, the signum function which usually reason to the problem of chattering and sensitivity to the noise of sensors. Secondly, the SMC may has unnecessarily great control signals to overcome all the parametric variations and disturbance input. To avoid these difficulties, for instance, one can use an approximate signum function, equivalent control (Low pass filter) and adaptation method with classic SMC**, Kaynak, et al., 2001**. The adaptation technique for finding the control gain $K(t)$ gives a minimum amount value of discontinuity gain causing in minimization of the chattering effect and control effort as clarified in the following section.

4. ADAPTIVE SLIDING MODE CONTROL

Consider the sliding mode controller with time varying gain $K(t)$ given by;

$$u(s,t) = -K(t)sign\big(s(x(t),t)\big) \quad (6)$$

Where u is the control input to be designed, K is the adaptation gain of control, x is the state vector, s is the sliding variable, and $sign(s)$ is the signum function modeled by

$$sign(s) \coloneqq \begin{cases} 1 & if\ s > 0 \\ -1 & if\ s < 0 \\ \in [-1;1] & if\ s = 0 \end{cases} \quad (7)$$

The adaptation law for the switching gain $K(t)$ is presented as follows**, Utkin,** and **Poznyak, 2013**; let ϵ being a positive parameter, then;

If $|s(x(t),t| > \epsilon > 0$, then $K(t)$ is the solution of the following differential equation

$$\dot{K}(t) = \alpha * |s(x(t), t)| \tag{8}$$

with > 0 , $K(0) > 0$, and ϵ is selected as a small positive constant.
If $|s(x(t), t| \leq \epsilon$, then $K(t)$ becomes

$$\begin{aligned} K(t) &= \beta|\eta(t)| + \gamma \\ \tau\dot{\eta}(t) + \eta(t) &= sign\big(s(x(t), t)\big) \end{aligned} \tag{9}$$

with

$$\beta = K(t^*) \tag{10}$$

where $\gamma > 0$, $\tau > 0$, and t^* is the largest value of t such that

$$|s\ (x(t^* - 0), t^* - 0)| > \epsilon\ , |s(t^*, t^*)| \leq \epsilon$$

Obviously, this controller is based on the real sliding mode concept**, Utkin,** and **Poznyak, 2013.**
By supposing that

$$|s\ (x(0)\ ,0)|\ > \epsilon$$

the ASMC law (Eqs. (8) & (9)) works as follows**, Utkin,** and **Poznyak, 2013**:

The adaptation law uses expanding the gain $K(t)$ to avoid limited uncertainty with unknown bounds in the system if the gain reaches to a value sufficiently huge after that the real sliding mode begins at $t = t_1$. In this time when the absolute of switching control is less than or equal, then $K(t)$ converts to another adaptation law (9) in this equation modify the gain with respect to the present uncertainties and perturbations. If the absolute of switching control is greater than ϵ because of the varying uncertainties and perturbations surpasses $\beta = K(t_1)$, then the real sliding mode will be pulverized and $K(t)$ returns to the first adaptation law (8) with the same steps. For more details about this control you can refer to**, Utkin,** and **Poznyak, 2013.**

5. SIMULATIONS RESULT AND DISCUSSIONS

A servo actuator system model with friction mechanism, which described in Eq. (1), is utilized here as an application to verify the effectiveness and robustness of the ASMC. The simulations are performed using MATLAB simulink fixed step solver OD4 (runge-kutta) with initial condition $e(0) = (\frac{\pi}{3600}\ ,0)$, as in **AlSamarraie, 2013.**

Two sets of simulation results are presented below. In the first set, the nominal system parameters were used while in the second set the parameter values are taken equal to 1.15 of nominal values i.e., each parameter value increased by 15% of its nominal value. In addition, the presence of external disturbance load is considered in the simulation process. For both simulation sets, the results of the present work are compared with the results obtained by using a conventional sliding mode controller. The conventional SMC design details for the servo actuator are found in Appendix A where the control input, u is as given by Eq. (A.10). Also, if $\lambda = 10$, and the reference angle and reference velocity as described in Eq. (5) and the adaptive SMC control input u read as in Eqs. (6), (7), (8), (9) and (10) for $\epsilon = 0.03$, $k(0) = 16$, $\alpha = 9300$, $\gamma = 5$ and $\tau = 10^{-3}$. Finally the system parameters and the external load used in the simulations are given in Table 1 and Table 2.

5.1 Simulation Results with Nominal System Parameters

In **Fig. 2** and **4** the time required to reach the desired angle and velocity respectively is less than 0.5 sec for both controllers (adaptive SMC and conventional SMC). This outcome is checked while plotting the error and maximum error of angle in **Fig. 3**, where it does not exceed 2.5×10^{-4} radian in position. The sliding variable s as shown in **Fig. 5** does not exceed ϵ after $7 * 10^{-3}$ sec for both controllers. The control efforts u for both controllers are clarified in **Fig. 7** while the adaptive gain $K(t)$ is plotted in **Fig. 6**.

As can be easily checked from **Fig. 5** and **7**, the ASMC forces the state to stay at the sliding manifold with 35 present less than control effort of conventional SMC. This is because the control gain $K(t)$ is smaller than that in the classical SMC where its value is decreased in the region $|s| < \epsilon$ according to the adaptation law in Eq. (9). In this region the control action is the equivalent one and sliding manifold is still attractive. As a result the performance of the ASMC resembles that of the classical SMC, as can be deduced from **Fig. 2, 3, 4, 5** and **8**. Moreover the chattering is attenuated since near the sliding manifold the ASMC is the equivalent control (**Fig. 7**). Attenuating system response chatter is clearly seen in **Fig. 5** and **8** when compared with classical SMC design. It must also be noted here that the ASMC designed without the need to know system parameters values and the bound on their values. The only information required to be known is that the variation on the system model uncertainty is bounded. The next simulation test examines the ASMC ability in the presence of system parameters uncertainty and unknown external disturbances.

5.2 Simulation Results with System Parameters which Increased by 15% of their Nominal Values and with the Presence of Disturbance Load $T_L = 2.5$.

In order to test the ability and performance of the ASMC the simulation is repeated here for the servo actuator system but with parameter values increased by 15% of their Nominal Values (for example $F_s = F_{s0} + 0.15 * F_{s0}$) and also with the presence of disturbance load $T_L = 2.5$. Table (2) presents the system parameters used in this set of simulation.

As in the first set of simulation the performance of the ASMC is similar to that for the classical SMC. This can be shown in **Fig 9, 10, 11** and **15**. Again **Fig. 13** and **14** prove that the control effort is decreased by 32 present by using ASMC when compared to the classical SMC as a direct result to the adaptation of $K(t)$ near the sliding manifold (**Fig. 13**). Finally **Fig. 12** and **15** shows that the chattering is attenuated as in the case of the nominal system model parameters.

6. CONCLUSIONS

The theory of adaptive sliding mode control is utilized here to design a robust sliding mode controller to the servo actuator system in the presence of model uncertainty and friction. The existence of a bound on the uncertainty of the system model is the only information required to be known when designing the proposed ASMC. The major feature of classical SMC, which is the insensitivity to model uncertainty and external disturbances during sliding motion, are preserved as can be deduced from the results obtained from the numerical simulations. The obtained results showed also that the ability and performance of the proposed control are similar to the case of using classical SMC. Additionally, the control efforts are smaller and the chattering amplitude is, consequently, attenuated as can be seen when comparing the simulation results with the results obtained using classical SMC.

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APPENDIX (A)

Consider the nonlinear uncertain system**, (**see for example **Utkin, et al., 2009)**

$$\left.\begin{aligned} e_1 &= e_2 \\ e_2 &= f_2(e) + g_2 u + d(t) \end{aligned}\right\} \tag{A.1}$$

where $e = [e_1\ e_2]^T$ is the state vector, u the control input to be designed and $d(t)$ is the disturbance term which includes the external load and friction. In terms of nominal and perturbation term Eq. $(\mathrm{A.1})$ can be rewritten by

$$\left.\begin{aligned} e_1 &= e_2 \\ e_2 &= f_{20}(e) + g_{20} u + \delta(e,u) \end{aligned}\right\} \tag{A.2}$$

Where $f_{20}(e)$ and $g_{20}(e)$ are the nominal functions of $f_2(e)$ and $g_2(e)$ respectively, while $\delta(e,u)$ is the uncertainty term results from the uncertainty in system dynamics and external load. The perturbation term $\delta(e,u)$ is given by

$$\delta(e,u) = \Delta g_2(e)u + \Delta f_2(e) + d(t) = \Delta\left(\frac{1}{J}\right)u - \left(\frac{1}{J}\right)(F + T_l) \tag{A.3}$$

Let the sliding variable selected as;

$$s = e_2 + \lambda e_1 \tag{A.4}$$

Then the sliding variable time derivative is

$$\dot{s} = \dot{e}_2 + \lambda \dot{e}_1 \qquad (A.5)$$

From Eq. (A.2) $\dot{s}$ is

$$\dot{s} = f_{20}(e) + g_{20}u + \delta(e,u) + \lambda e_2 \qquad (A.6)$$

To design a sliding mode controller, the elected Lyapunov function is

$$V = |s| \qquad (A.7)$$

And its time derivative $\dot{V}$ is

$$\dot{V} = sgn(s) * \dot{s} \qquad (A.8)$$

Or

$$\dot{V} = sgn(s) * (f_{20}(e) + g_{20}u + \delta(e,u) + \lambda e_2) \qquad (A.9)$$

The request is the selection of u such that $\dot{V}$ is negative definite. In this work u is chosen as in the traditional sliding mode by

$$u = \frac{1}{g_{20}(e)}\left(-f_{20}(e) - \lambda e_2 - K(e) * sign(s)\right) \qquad (A.10)$$

then $\dot{V}$ becomes;

$$\begin{aligned} \dot{V} &= sign(s) * \left(-K(e) * sign(s) + \delta(e,u)\right) \\ &= -K(e) + sign(s) * \delta(e,u) \\ &\le -K(e) + |\delta(e,u)| \end{aligned} \qquad (A.11)$$

The gain $K(e)$ that will make the inequality (A.11) less than zero (attractiveness of the sliding manifold and sliding motion) is selected as follows;

$$K(e) > |\delta(e,u)| \qquad (A.12)$$

Where

$$\begin{aligned} |\delta(e,u)| &= |\Delta f_2(e) + d(t)| + |\Delta g_2(e)u| \\ &= |\Delta f_2(e) + d(t)| + \left|\Delta g_2 * \{\frac{1}{g_{20}(x)}(-f_{20}(e) - \lambda e_2 - K(e) * sign(s))\}\right| \end{aligned}$$

$$= |\Delta f_2(e) + d(t)| + \left|\frac{\Delta g_2(e)}{g_{20}(e)}\right| * |f_{20}(e) + \lambda e_2| + \left|\frac{\Delta g_2(e)}{g_{20}(e)}\right| * K(e) \qquad (A.13)$$

When substituting Eq. (A.13) in the inequality (A.12) and resolve for the gain $K(e)$ to obtain the following inequality:

$$K(e) > \frac{\max\{|\Delta f_2(e) + d(t)| + \left|\frac{\Delta g_2(e)}{g_{20}(e)}\right| * (|f_{20}(e) + \lambda e_2|)\}}{1 - \max\left(\left|\frac{\Delta g_2(e)}{g_{20}(e)}\right|\right)}$$

Now let $K(e)$ equal to:

$$K(e) = k_0 + \frac{\max\{|\Delta f_2(e)| + \left|\frac{\Delta g_2(e)}{g_{20}(e)}\right| * (|f_{20}(e) + \lambda e_2|)\}}{1 - \max\left(\left|\frac{\Delta g_2(e)}{g_{20}(e)}\right|\right)} \qquad (A.16)$$

where

$$k_0 > 0$$

$$\begin{aligned} |\Delta f_2(e) + d(t)| &= \left|\frac{1}{J}(F + T_L)\right| \\ &\leq \left|\tfrac{1}{J}(F))\right| + \left|\tfrac{1}{J}(T_L)\right| = \left|\tfrac{1}{J}(F_c + F_s + \sigma|\dot{x}|)\right| + \left|\tfrac{1}{J}(T_L)\right| \\ &\leq \left|\tfrac{1}{J_{min}}(F_{cmax} + F_{smax} + \sigma_{max}|\dot{x}|)\right| + \left|\tfrac{1}{J_{min}}(T_{Lmax})\right| \end{aligned}$$

$$|\Delta g_2(e)| = \left|\frac{1}{\Delta J_{min}}\right|.$$

The value of $k_0 = 0.25$ and the classical SMC parameters with maximum uncertainty (25%) are presented in the following Table A.1.

Table 1. DC servo actuator and friction model nominal parameters**, Xie, 2007.**

Nominal Parameters	Definition	Value	Unit
J_o	inertia of moment	0.2	Kgm^2
F_{so}	Stiction friction	2.19	Nm
F_{co}	Coulomb friction	16.69	Nm
$\dot{x}_{so}$	Stribeck velocity	0.01	rad/sec
σ_o	viscous friction coefficient	0.65	Nm.sec/rad
T_{Lo}	the load torque	2	Nm

Table 2. DC servo actuator and friction model parameters used in the simulations.

Parameters	Definition	Value	Unit
J	inertia of moment	0.23	Kgm^2
F_s	Stiction friction	2.5185	Nm
F_c	Coulomb friction	19.1935	Nm
$\dot{x}_s$	Stribeck velocity	0.0115	rad/sec
σ	viscous friction coefficient	0.7475	Nm.sec/rad
T_L	the load torque	2.5	Nm

Table A.1. System parameter with 25% uncertainty used for Classical SMC.

Parameters	Definition	Value	Unit
J_{min}	Minimum value of inertia of moment	0.15	Kgm^2
$F_{s\,max}$	maximum value of Stiction friction	2.7375	Nm
$F_{c\,max}$	maximum value of Coulomb friction	20.8625	Nm
σ_{max}	maximum value of viscous friction coefficient	0.8125	Nm.sec/rad
$T_{L\,max}$	maximum value of the load torque	2.5	Nm

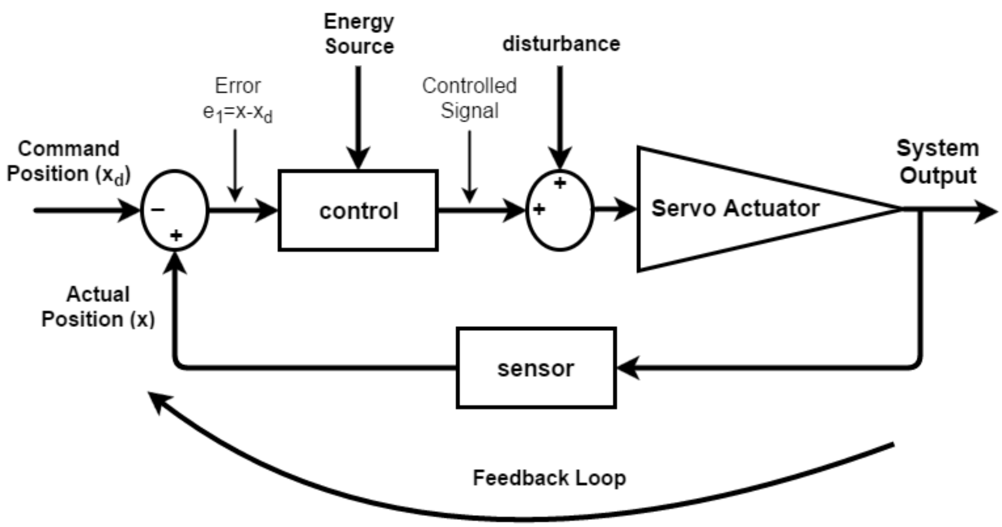

Figure 1. Closed loop position servo actuator model.

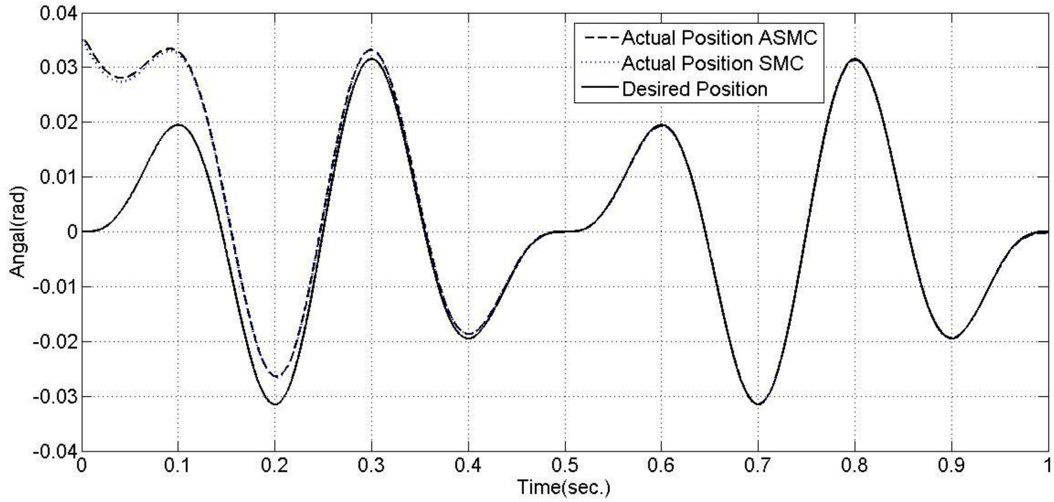

Figure 2. Angle vs. time for the servo actuator.

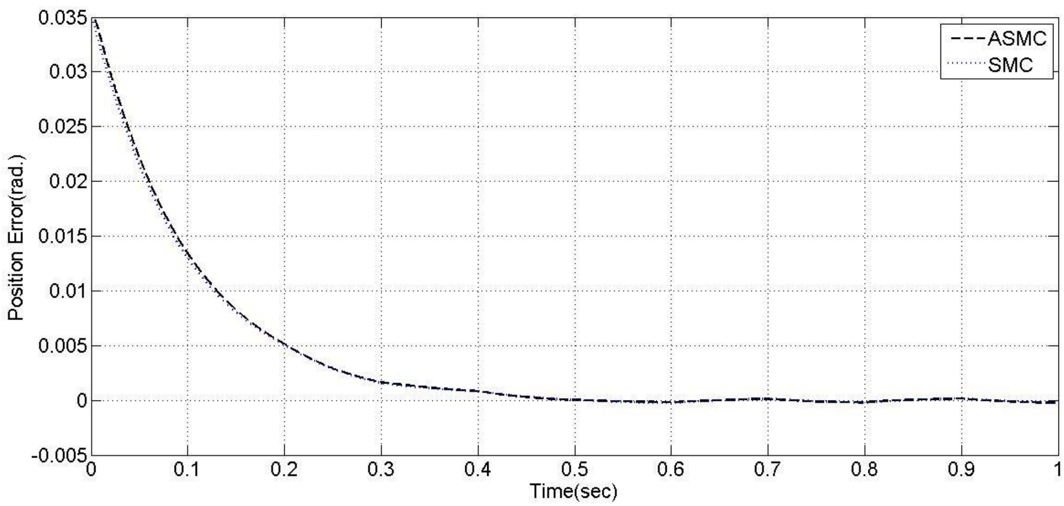

Figure 3. The position error vs. time for the servo actuator.

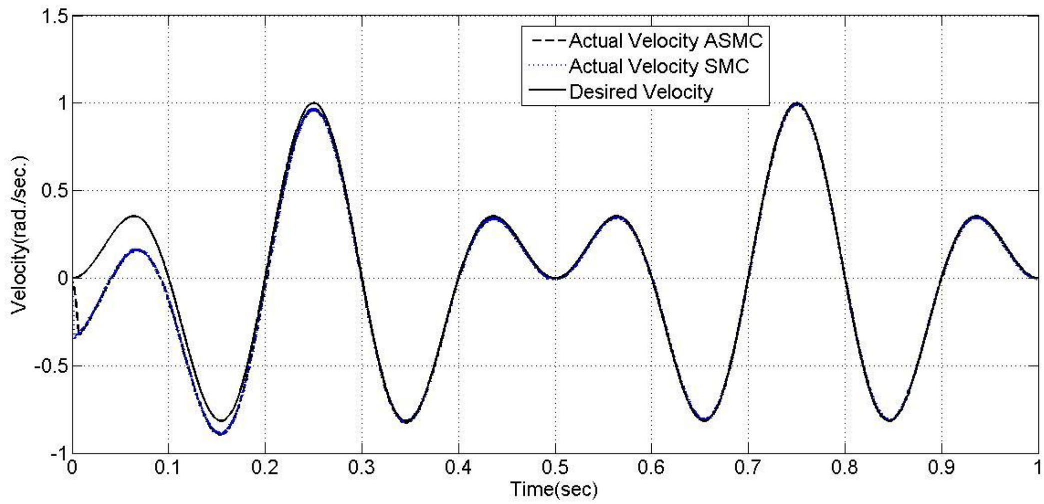

Figure 4. Velocity vs. time for the servo actuator.

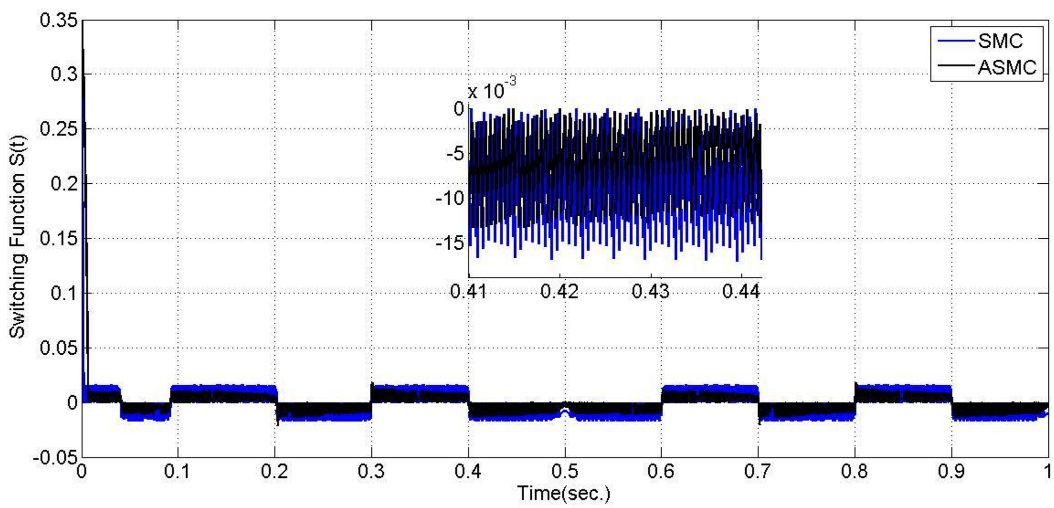

Figure 5. Sliding variable $s(x)$ vs. time for the servo actuator.

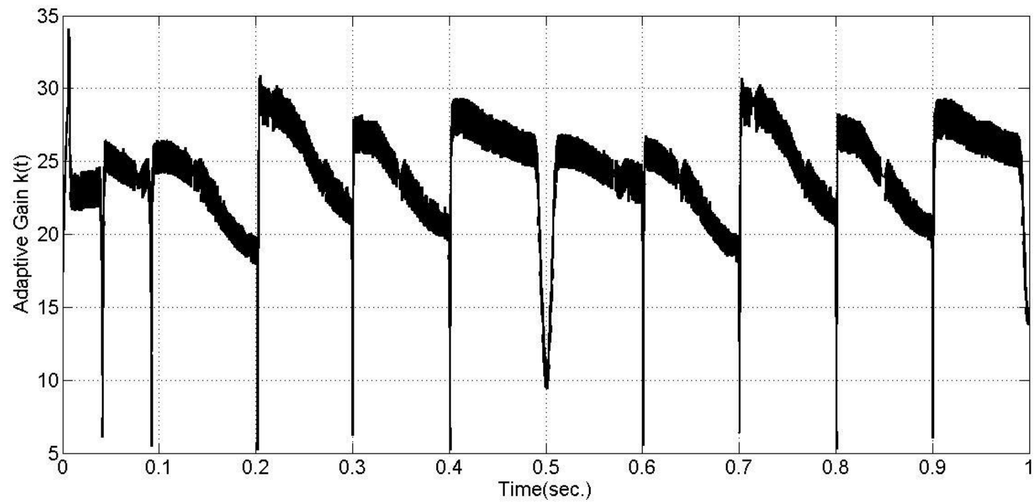

Figure 6. Adaptive gain $K(t)$ vs. time for the servo actuator.

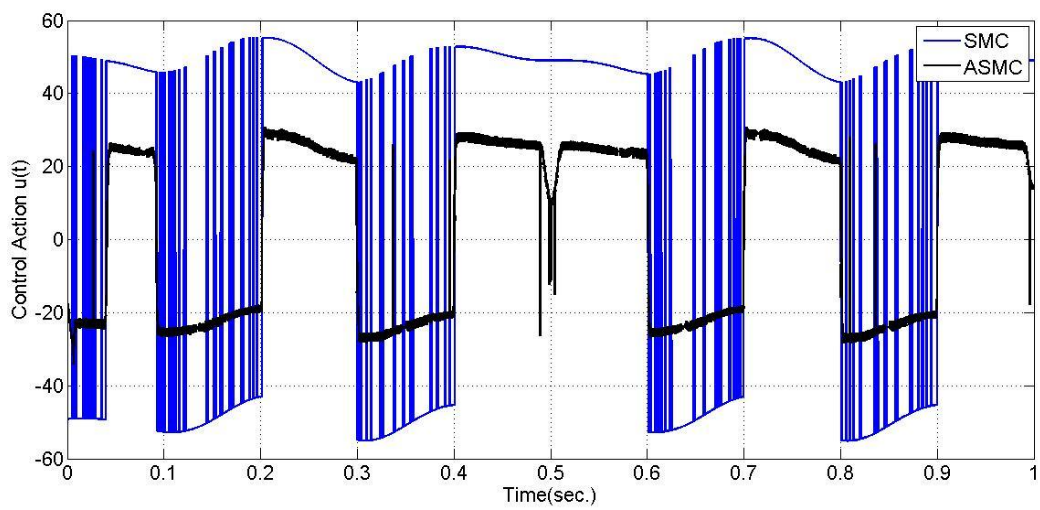

Figure 7. Control input u vs. time for the servo actuator.

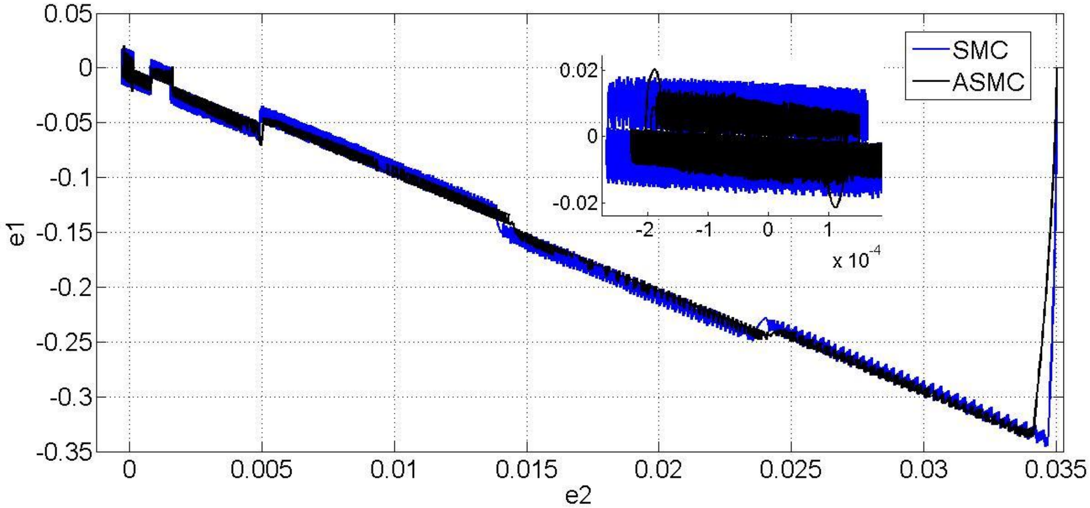

Figure 8. Phase plot of e2 vs. e1 for the servo actuator.

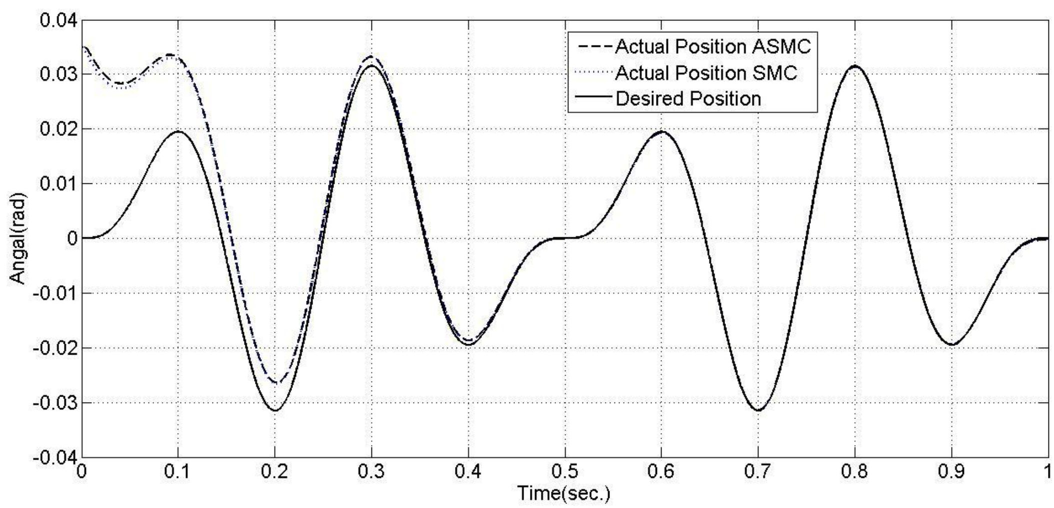

Figure 9. Angle vs. time for the servo actuator with parameter uncertainty and disturbance load.

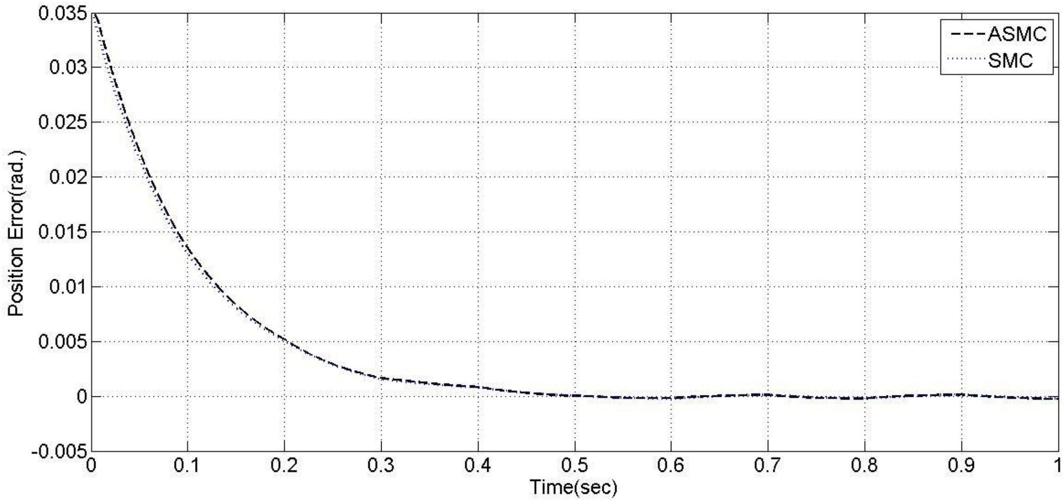

Figure 10. The position error vs. time for the servo actuator with parameter uncertainty and disturbance load.

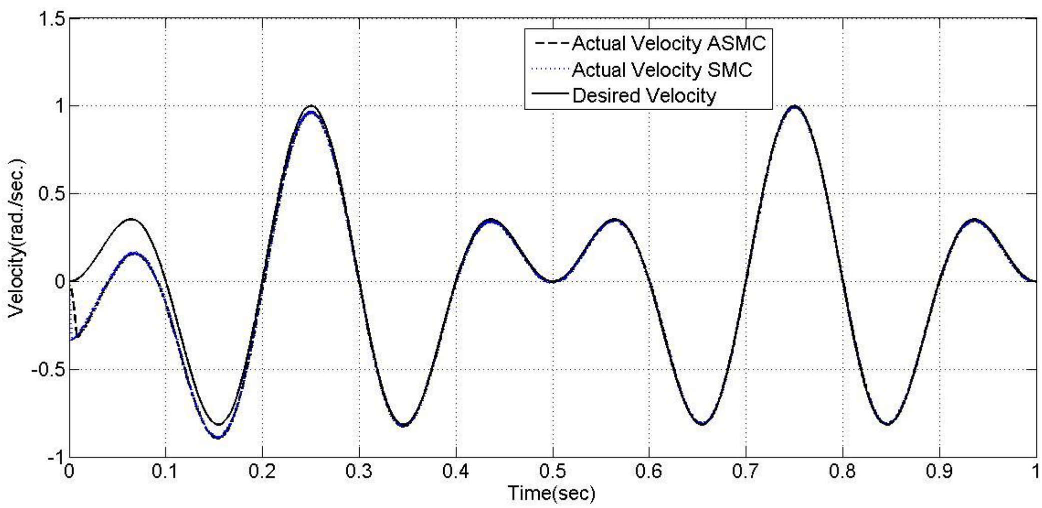

Figure 11. Velocity vs. time for the servo actuator with parameter uncertainty and disturbance load.

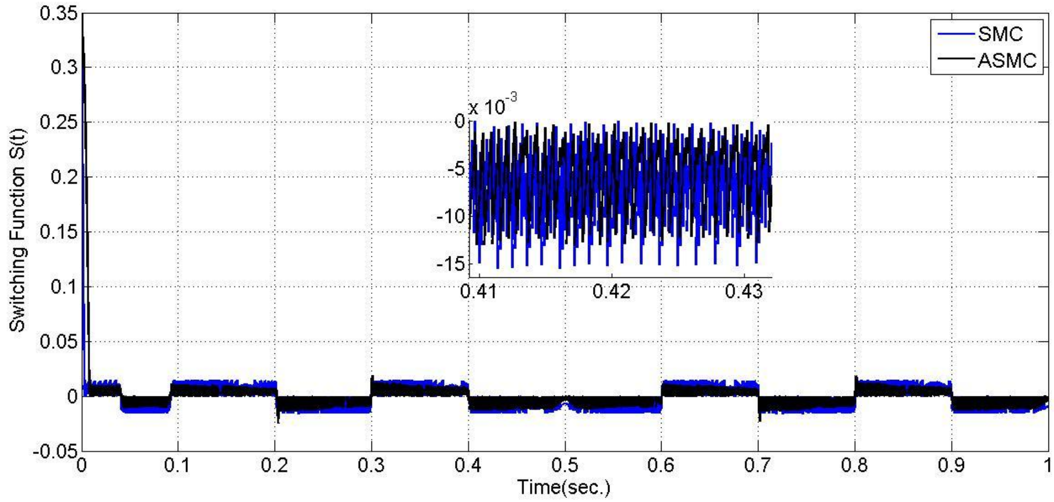

Figure 12. Sliding variable $s(x)$ vs. time for the servo actuator with parameter uncertainty and disturbance load.

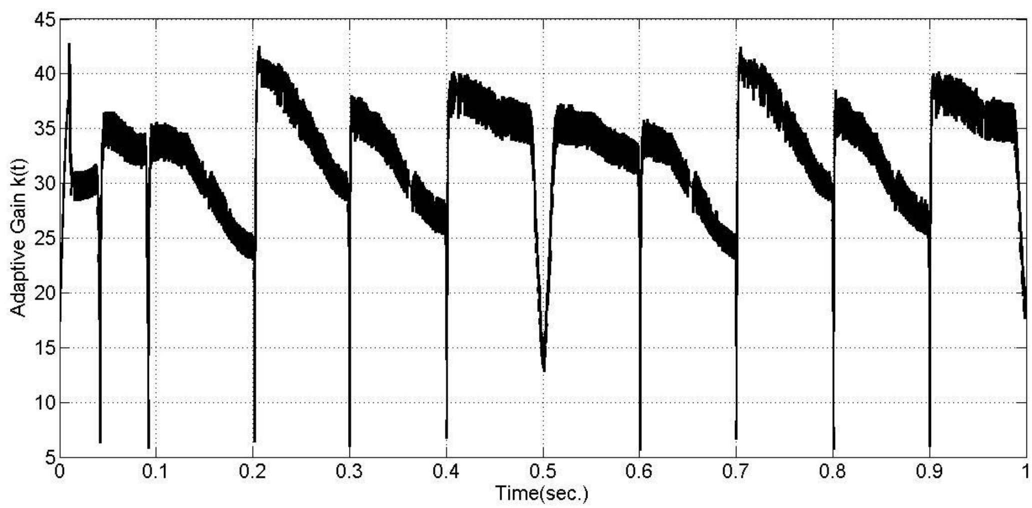

Figure 13. Adaptive gain $K(t)$ vs. time for the servo actuator with parameter uncertainty and disturbance load.

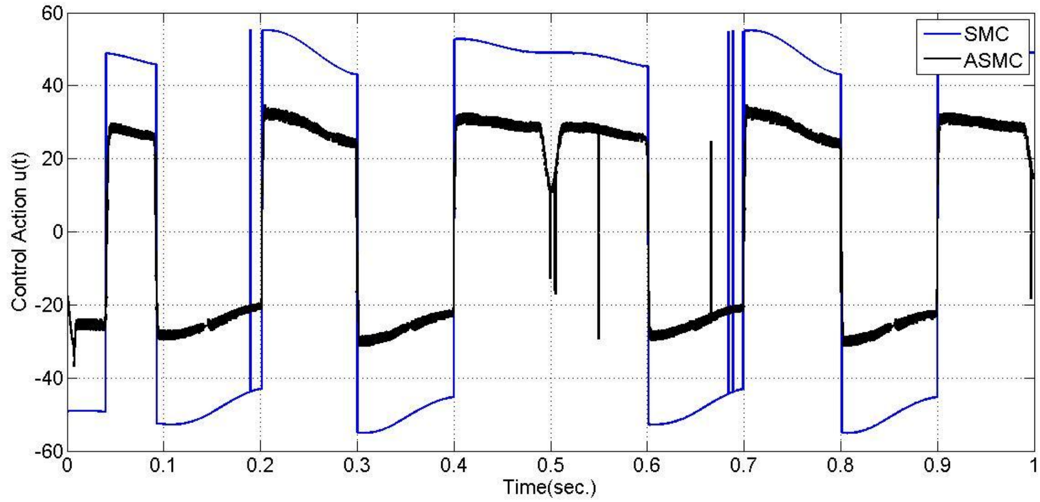

Figure 14. Control input u vs. time for the servo actuator with parameter uncertainty and disturbance load.

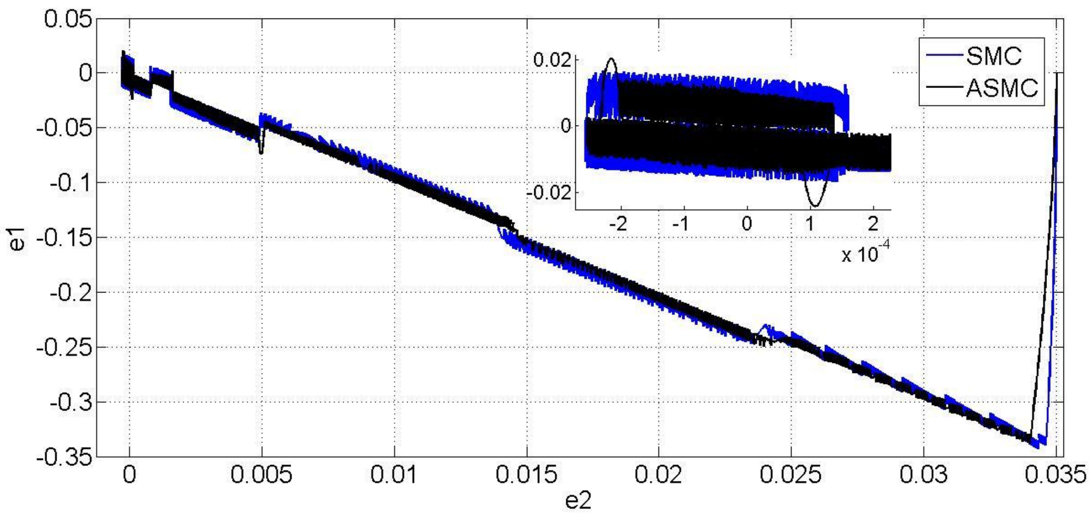

Figure 15. Phase plot of e2 vs. e1 for the servo actuator with parameter uncertainty and disturbance load.